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Short-Term Reversals and Longer-Term Momentum Around the World: Theory and Evidence

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Abstract

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Stock returns exhibit reversals at short horizons but slowly transition to momentum over longer horizons. To help understand this pattern, we develop a dynamic model with short- and long-horizon noise traders, informed investors, and uninformed investors who underreact to information they do not themselves produce. The model accords with the transition from reversals to momentum, and yields the following novel predictions: (i) attenuated reversals following earnings announcements, (ii) a negative relation between momentum and reversal profits across economies and time, and (iii) larger reversals when noise trading is more volatile. Empirical analysis using U.S. and international data supports these predictions.

Finance academics have long focused on tests of stock return predictability. A particularly simple form of such a test is to consider whether past stock returns forecast future returns. For example, Table 2 of our paper (discussed in detail later) reports the results of a Fama and MacBeth (1973)-style regression of monthly stock returns on several lags of these returns, for U.S. as well as international stocks. The table, in essence, extends Table I in Jegadeesh (1990) to a more recent time-period and to the global context. The literature has examined two phenomena that are evident in this table. First, the one-month lagged coefficients are negative and significant, pointing to the familiar monthly reversals (highlighted in Jegadeesh (1990)). In addition, the coefficients at lags of three to 12 months are positive and significant, indicating momentum, which has been extensively studied by Jegadeesh and Titman (1993), Rouwenhorst (1998), and others. We also observe that the second lag is insignificant, suggesting a gradual transition from reversals to momentum.

This paper provides a unified model that addresses the above pattern of return predictability, and yields novel predictions that we test with U.S. and international data. Specifically, we consider a setting with three rounds of trade. In each round, a signal becomes available about a long-term fundamental, such as an annual report. We interpret the signal in the intermediate round as an earnings announcement, and the other two signals as analysts' guidance, or other similar signals produced by market professionals. The model includes short- and long-horizon noise traders, and two types of active investors. The first type, the informed, observe signals about fundamentals, and are neoclassical utility-maximizers as in Grossman and Stiglitz (1980). The second type, the uninformed, underreact to information they do not themselves produce, due to a form of overconfidence (Odean (1998) and Luo, Subrahmanyam, and Titman (2021)). This underreaction creates momentum;[1] however, reversals also arise because risk averse investors require a premium to absorb noise trades. The interactions between these phenomena in our model result in short-term reversals and longer-term momentum, as well as the low predictability in between.[2] A calibration exercise shows that under reasonable conditions, theoretical magnitudes accord with empirical levels of reversal and momentum profits.

[1] Chan, Jegadeesh, and Lakonishok (1996) provide evidence that momentum arises from underreaction to information such as earnings announcements and analysts' revisions.

[2] While even longer-term reversals at horizons of two years or more (De Bondt and Thaler (1985), Zaremba (2016)) are not the focus of this paper, we discuss them in Section 5.

We proceed to test the model's new predictions. Our first prediction is that short-term reversals should attenuate after earnings announcements. Intuitively, because the market receives more information during earnings announcements than in other periods, a stronger price underreaction after these announcements weakens the reversals. We find that this is indeed the case, in both U.S. and international samples. Next, because more noise trading implies stronger reversals and attenuated momentum, the model predicts an inverse relation between momentum and reversal profits. We show that these profits are indeed negatively correlated both across countries, and across time within countries.[3]

We explore whether cross-country differences in momentum and reversal profits are related to differences in cultural traits, in ways suggested by our model. Specifically, we consider the attributes introduced by Hofstede (1991) that have been related to stock returns in earlier literature. These include individualism, considered by Chui, Titman, and Wei (2010) and uncertainty avoidance, discussed in Nguyen and Truong (2013). Consistent with Chui, Titman, and Wei (2010), individualism is significantly related to momentum profits, but not to reversals. Uncertainty avoidance, however, is related to both. Our explanation, consistent with Nguyen and Truong (2013), is that cultures exhibiting lesser avoidance of uncertainty are less conservative, and thus more likely to trade on long-run fundamentals. In turn, we propose that investors from low uncertainty-avoiding countries may have more fundamental traders relative to noise traders, which, according to our model, enhances momentum and attenuates reversals.

Another implication of the model is that increased intensity of noise traders' order flows leads to stronger reversals. We proxy for noise traders by retail investors, using the Boehmer, Jones, Zhang, and Zhang (2021) method to identify retail trades. We measure the trading intensity of retail investors both by their absolute monthly order imbalance, and the monthly standard deviation of their order imbalance. We find that reversals are indeed stronger when retail imbalance is higher in absolute terms or is more volatile.

Our model provides theoretical insights beyond the empirical motivation. For example, we show that short-term reversals are amplified by sequential resolution of uncer-

[3]A competing hypothesis is that the main source of the correlation between momentum and reversals across countries is variations in arbitrage capital, but this would suggest a positive correlation between momentum and short-term reversal profits (more arbitrage capital should reduce both forms of predictability), and this is the opposite to what we find.

tainty. Thus, if positive noise trades move prices above fundamentals this month, we expect to see a partial reversal towards fundamentals due to the new information signal next month. This phenomenon is a consequence of risk aversion and occurs regardless of underreaction. Thus, without underreaction, noise trades and sequential releases of information act in the same direction: they both accentuate reversals. The underreaction to the information signals, however, opposes the reversals. We find that such underreaction can more than offset reversals at longer horizons, but not at shorter horizons, only if a large enough proportion of noise traders liquidate their positions quickly.[4] It follows that momentum and short-term reversals occur together in a parameter range where there are upper and lower bounds on noise trading (which ensure momentum and reversals, respectively), as well as a lower bound on the proportion of noise traders with short horizons.

We demonstrate that the precision of information signals is a key parameter governing whether underreaction offsets reversals due to noise trades. To understand the intuition, note that if a signal is completely uninformative (imprecise), investors do not react to the signal at all, and noise-trader-induced reversals remain unaffected in this case. In the less extreme case where precision of the earnings announcement is not too low, underreaction does mitigate reversals.

Prior research has considered different variations of the forces in our model. In some models, return persistence occurs if investors underassess the precision of either their own or others' information signals, or equivalently, the information content of prices; see Banerjee (2011), Eyster, Rabin, and Vayanos (2019), Mondria, Vives, and Yang (2022), and Luo, Subrahmanyam, and Titman (2021). There also are models which illustrate the general principle that noise traders (informed traders) generate reversals (continuations) conditional on their trades (for examples, see Glosten and Milgrom (1985), Campbell, Grossman, and Wang (1993), Holden and Subrahmanyam (2002), Llorente, Michaely, Saar, and Wang (2002), and Albuquerque and Miao (2014)). Our paper contributes to the above literature by presenting an integrated model that sheds light on the transition from short-term reversals to longer-term momentum, and makes additional predictions for which

[4] While even longer-term reversals at horizons of two years or more (De Bondt and Thaler (1985), Zaremba (2016)) are not the focus of this paper, we discuss them in Section 5.

we find empirical support.

There are other approaches to explaining momentum. For instance, Grinblatt and Han (2005) and Da, Gurun, and Warachka (2014) respectively generate underreaction and momentum via the disposition effect, and the idea that investors pay less attention to news that arrives gradually, as opposed to discrete chunks.[5] Bordalo, Gennaioli, Ma, and Shleifer (2020) generate momentum via extrapolative expectations. We conjecture that our basic results trading off news-induced momentum against noise-trader-induced reversals would go through under these alternative rationales.[6]

We also contribute to the empirical literature on short-term reversals.[7] Explanations for such reversals have typically focused on the supply side of liquidity. For example, Hameed and Mian (2015) show that reversals are stronger after market declines, which is when capital constraints and risk aversion of liquidity providers are likely to be more relevant. Further, Cheng, Hameed, Subrahmanyam, and Titman (2017) find that short-term reversals are higher following declines in the number of active institutional investors, who provide liquidity to retail investors. Our empirical work complements these studies by emphasizing the demand side of liquidity. We find that reversals are exacerbated when absolute retail order flows are high and thus provide direct evidence that short-term reversals are indeed driven by unsophisticated investors, who are more likely to be noise traders. In addition, we provide new evidence on how and why momentum and

[5] Barardehi, Bogousslavsky, and Muravyev (2022) indicate that momentum profits primarily emanate from prices during normal trading periods, as opposite to overnight hours, which is consistent with the view that momentum arises because the trades of investors underreact to cash flow information (more likely to be released during the trading day). Huang (2022) shows that the spread between winner and loser returns in the portfolio formation period is negatively related to momentum profits, which is consistent with our model if a small spread indicates more underreaction to information within the formation period (and thus more price movement post-formation).

[6] Some models explain the empirically observed return predictability from rational perspectives. Cujean and Hasler (2017) show that in bad times, investors' opinions polarize due to increasing uncertainty, and the persistent disagreement causes momentum. Johnson (2002) shows that the momentum effect can arise in a model with rational investors when expected dividend growth rates vary over time. Though these papers provide important economic insights, the Sharpe ratios achievable via momentum seem too large to be explained by rational models (Brennan, Chordia, and Subrahmanyam (1998)).

[7] Jegadeesh (1990) and Lehmann (1990) consider reversals at monthly and weekly horizons, respectively, and our study mainly focuses on reversals at the former horizon. There is a separate literature focusing on even shorter (daily) horizons. Thus, Baltussen, van Bekkum, and Da (2019) and Da, Tang, Tao, and Yang (2023) attribute daily reversals in stock and commodity indices to noise trading. Nagel (2012) and So and Wang (2014) find that daily individual stock reversals are higher when market makers are financially constrained or face greater inventory risks. Cakici and Zaremba (2022) show that part of these reversals can be explained by salience, i.e., how extreme the return is relative to that for the average stock on a given day.

reversal profits covary across countries.

Note that the tradeoff between the intensity of noise trading (which generates reversals) and fundamental information (underreaction to which generates momentum) determines whether returns exhibit momentum or reversals in our model. Recent empirical work supports the role of these forces in the prevalence of momentum and reversals across countries. For example, Du et al. (2023) and Hameed, Ni, and Tan (2023) find no unconditional momentum in China and Singapore, respectively, but significant momentum when they exclude low-priced small cap stocks, which have bigger retail ownership than the other stocks. George, Hwang, and Li (2023) further document that there is momentum in China excluding February, i.e., the onset of the Chinese New Year, when retail investors are the most active. Chui, Ranganathan, Rohit, and Veeraraghavan (2023) find that a 2010 policy change that increased free float in India increased momentum profits. Given that the policy shift was accompanied by a significant increase in institutional holdings (Jawed and Kotha (2020)), the finding supports our idea that momentum arises from fundamentals-based institutional trades.[8]

The rest of this paper is organized as follows. In Section 1, we present the general theoretical framework. In Section 2, we use an analytic solution for a special case to obtain intuition. In Section 3, we use numerical analysis to study the general case. In Section 4, we empirically test some implications of our model. Section 5 considers possible extensions of our approach. Section 6 concludes. All proofs, unless otherwise stated, appear in Appendix A.

1 The Setting

We now present the structure of our model which includes utility-maximizing informed and uninformed investors, as well as noise traders. As we explain below, we assume that uninformed investors underreact to information due to a form of overconfidence. Throughout the paper, unless otherwise specified, all individual random variables are

[8] Lou (2012) and Vayanos and Woolley (2013) also relate serially correlated institutional fund flows to return momentum. Chui, Subrahmanyam, and Titman (2022) show that Chinese B shares exhibit momentum but not short-term reversals, while A shares demonstrate the opposite pattern. They attribute this to a higher prevalence of institutions in B shares, using a simple model to motivate their analysis. A caveat is that the sample of firms that issue both A and B shares is small (less than ninety), thus limiting the generality of conclusions that can be drawn from their study.

mutually independent, and normally distributed with zero mean. The variance of a generic random variable x is denoted by ν_x.

Assets: There is a risky stock in zero net supply. This security is traded at Dates 0, 1, 2, and 3, and then liquidated at Date 4. Its liquidation value is given by $V = \theta$. We interpret θ to be an annual report. There is also a risk-free asset whose price and gross return are each set to unity.

Investors: A unit mass of active investors, each indexed by i, derive utility from their final wealth and seek to maximize the following standard exponential utility function:

$$U(W_{i4}) = -\exp(-AW_{i4}),$$

where W_{i4} is the investor's final wealth, and A is a positive constant representing the absolute risk aversion coefficient.

In addition to the demand for shares from active traders, at each date t ($t = 1$, 2, or 3), there is a new demand z_t from noise traders, which is drawn from a normal distribution with mean zero and variance ν_{z_t}. We model noise traders with different horizons as follows. A fraction of the date t noise demand, $(1-\mu)z_t$, is unwound at Date $t+1$; the rest of this demand, μz_t, is unwound at Date $t+2$. $\mu \in [0,1]$ is a constant parameter. The date 3 demand, of course, is fully rewound at Date 4. Taken together, the net noise demand equals z_1 $(z_2 + \mu z_1)$ $(z_3 + \mu z_2)$ at Date 1 (2) (3).[9]

Information and Beliefs: Date 0 is the starting date and is used to determine an initial price. At Date 1, a public signal $f = \theta + \xi + \epsilon + \zeta$ is revealed, where ξ (ϵ) (ζ) is drawn from a normal distribution with mean zero and variance ν_ξ (ν_ϵ) (ν_ζ). At Date 2, a second public signal $F = \theta + \xi + \epsilon$ is revealed. At Date 3, a mass λ of "informed" active investors observe a private signal $s = \theta + \xi$, which is a refined version of the public signals. The remaining mass $1 - \lambda$ of "uninformed" active investors do not observe s. $\lambda \in [0,1]$ is a constant parameter. We interpret the Date 2 signal as corresponding to an earnings announcement, whereas the Dates 1 and 3 signals can be interpreted as analysts' forecasts or guidance, or other information produced about the annual report.

[9]It is possible to model a more general form of the Date t demand as having a component that is reversed at horizons longer than $t+2$, but our analysis indicates that such an assumption does not lead to analytic solutions. We address this issue numerically in Section 3.4.

Informed investors are neoclassical utility maximizers with rational expectations, as in Grossman and Stiglitz (1980). We model underreaction by assuming that uninformed active investors exhibit a form of overconfidence that makes them skeptical about information which they do not themselves produce (Odean (1998), Luo, Subrahmanyam, and Titman (2021)). Thus, they believe s reveals only part of the payoff θ. Specifically, assuming the payoff θ can be decomposed into two independent components, that is, $\theta = \theta_1 + \theta_2$, uninformed investors believe that $s = \theta_1 + \xi$, so s reveals only the component θ_1. Further, from the uninformed's perspective, $f = \theta_1 + \xi + \epsilon + \zeta$ and $F = \theta_1 + \xi + \epsilon$, so f and F also reveal only the component θ_1. For example, the uninformed may believe the informed (and managers) are good at assessing competition for the firm's products, but may be skeptical about their ability to forecast deeper elements, such as market conditions for downstream customers (Cohen and Frazzini (2008)) or the impact of less-visible competitors (Baik, Hoberg, Kim, and Oh (2017)). We assume that θ_1 (θ_2) follows a normal distribution with mean zero and variance $\nu_{\theta_1} = \kappa^{-1}\nu_\theta$ ($\nu_{\theta_2} = (1 - \kappa^{-1})\nu_\theta$), where $1 \leq \kappa < \infty$. The parameter κ then represents the scale of underreaction.

The sequence of public and private signals allows us to obtain analytic solutions, since the fixed-point setting with asymmetric information (Grossman and Stiglitz (1980)) has to be solved only at Date 3 when the private signal is revealed. The presence of the privately-informed makes the modeling more complete, in that we are able to demonstrate that our results obtain in a specification with rational agents and asymmetric information. However, the general intuition requires only that some investors underreact to signals at each date. Thus, the reasoning goes through when the uninformed underassess the precision of information signals, or fail to condition on market prices (Hong and Stein (1999), Eyster, Rabin, and Vayanos (2019)), or if the informed receive private signals at each date. Our exploratory investigation reveals that such models are complex without an analytic solution even for special cases, though they deliver similar results numerically.[10] Also, while our main model has a finite horizon, in Appendix B we show that our model can be interpreted as applying to an infinite horizon, with a nonzero mean for the final payoff V and a nontrivial riskfree rate.

[10] Adding the traditional form of overconfidence, in which the informed investors over-assess their signal precision (Daniel, Hirshleifer, and Subrahmanyam (1998)), also makes the model less tractable. We briefly revisit this point in Section 5.2.

Table 1 describes the timeline of our model. Note that the Date 4 price P_4 equals the final payoff θ. We can solve for the other prices using backward induction. The following proposition describes the forms of the prices that obtain:

Proposition 1 *The prices at Dates 1, 2, and 3 take the following forms:*

$$\begin{aligned} P_3 &= \gamma_1 \omega + \gamma_2 F + \gamma_3 \mu z_2, \\ P_2 &= \beta_1 F + \gamma_3 \mu z_2 + \beta_2 (z_2 + \mu z_1) - \beta_3 z_2, \\ P_1 &= \alpha_1 f + \beta_2 \mu z_1 + \alpha_2 z_1, \end{aligned}$$

where $\omega \equiv s + \delta z_3$, and the α's, β's, γ's, and δ are constants. The Date-0 price P_0 takes the normalized value of zero.

The Date-3 price P_3 only partially reveals s because of the noise trade z_3 (via ω). P_3 is also related to the earnings announcement F because when trading at Date 3, uninformed investors use it as an additional information source. Note that if $\mu > 0$ the Date-2 noise demand z_2 does not unwind completely at Date 3; but the unwound portion μz_2 still influences P_3 because of the limited risk-bearing capacity of the market.

Considering the expression for the Date-2 price, P_2, we see that the first term $\beta_1 F$ depends on the expectation of the final payoff θ, conditional on F. The second term $\gamma_3 \mu z_2$ is a speculative component that arises because active investors anticipate that if $\mu > 0$, then the noise demand z_2 will not unwind completely at the next date and will still influence P_3. The third term $\beta_2(z_2 + \mu z_1)$ is due to the current net noise demand $z_2 + \mu z_1$; when this noise demand is positive (negative), it pushes P_2 up (down). The last term, $-\beta_3 z_2$, represents the influence of the anticipated effect of z_2 on the next period's price (P_3), which reflects the fact that z_2 is only partially unwound at Date 3.

The Date-1 price P_1 takes a similar form as price P_2. The first term $\alpha_1 f$ depends mainly on the expectation of the final payoff θ, conditional on the public signal f. The price P_1 also includes a term related to investors' anticipation that the unwound noise demand z_1 still affects price P_2 at the next date (i.e., $\beta_2 \mu z_1$), and a term ($\alpha_2 z_1$) that combines the direct effect of the net noise demand as well as the indirect effect coming from investors' anticipation of the unwinding of the noise trades.

Our analysis focuses on two measures of return predictability. The first is a short-term predictability measure, which is return predictability from past returns in adjacent periods. The second is a measure of long-term predictability, which is the predictability over time spans that are greater than one period.[11] More explicitly, the short-term predictability measure is expressed as the average of the three contiguous covariances

$$\mathcal{S} = \frac{\sum_{t=1}^{3} \operatorname{Cov}(P_t - P_{t-1}, P_{t+1} - P_t)}{3}, \tag{1}$$

and the long-term predictability measure is expressed as

$$\mathcal{L} = \operatorname{Cov}(P_2 - P_0, P_4 - P_2). \tag{2}$$

2 An Analytical Solution for a Special Case

In the most general case, the predictability measures $\mathcal{S}$ and $\mathcal{L}$ cannot be analyzed in closed form. However, we now provide analytical results for a special case where we let $\lambda = 0$ and assume that uninformed investors directly learn the signal s. This represents the limit of the case where $\lambda \to 0$ so that the mass of uninformed relative to informed investors is large. For simplicity, we also fix the Date-3 noise trade $z_3 \equiv 0$, so that the Date-3 price fully reveals informed investors' private signal s. We also assume that the noise trades at Dates 1 and 2 have the same scale (i.e., $\nu_{z_1} = \nu_{z_2} = \nu_z > 0$). Further, we assume that $0 \leq \mu \leq 1$; in other words, that new noise trader positions are reversed over at most the next two periods.

We use the simplified model above to consider in closed-form the interaction between momentum and short-term reversals, as well as how return predictability is influenced by public announcements and how serially dependent noise trades influence price patterns. We calibrate the general model to match the empirical magnitudes of short-term reversals and longer-term momentum; Section 3 provides details of this exercise, and performs additional comparative statics.

Let $\kappa_s = \nu_{\theta_1} + \nu_\xi$, $\kappa_F = \kappa_s + \nu_\epsilon$, and $\kappa_f = \kappa_F + \nu_\zeta$, and let the subscript ℓ indicate that the expectation is based on uninformed beliefs, as described in the previous section. The following lemma presents the prices in analytic form.

[11] We use return autodependence to measure reversals and momentum. Luo, Subrahmanyam, and Titman (2021) show that the autocovariance represents the average profit of a cross-sectional portfolio strategy (see also Lehmann (1990)).

Lemma 1 *In the special case, the prices are as follows:*

$$P_3 = E_\ell(\theta|s) + c_3\mu z_2, \tag{3}$$
$$P_2 = E_\ell(\theta|F) + c_3\mu z_2 + b_2(z_2 + \mu z_1), \tag{4}$$
$$P_1 = E_\ell(\theta|f) + b_2\mu z_1 + a z_1, \tag{5}$$

where

$$c_3 = A(\nu_\theta - \nu_{\theta_1}^2\kappa_s^{-1}), \quad b_2 = (\nu_{\theta_1}^2/\kappa_s)(1-\kappa_s\kappa_F^{-1}), \quad a \equiv a_1 - a_2, \quad \textit{with}$$
$$a_1 \equiv A\frac{\nu_{\theta_1}^2}{\kappa_F}\left(1-\frac{\kappa_F}{\kappa_f}\right) + A\frac{(b_2+c_3\mu)^2}{\nu_z^{-1}+Ab_2+Ac_3\mu^2}, \quad \textit{and} \quad a_2 \equiv A\frac{(b_2+c_3\mu)\,b_2}{\nu_z^{-1}+Ab_2+Ac_3\mu^2}\mu.$$

P_0 *takes the normalized value of zero.*

Noting that $a \equiv a_1 - a_2$, the noise demand z_1 that arises at Date 1 affects P_1 through a direct demand effect (i.e., $a_1 z_1$) and an indirect effect $-a_2 z_1$ due to the anticipated date 2 noise trade of μz_1.[12] Of course, at Date 2, the unwound μz_1 influences P_2. At Date 3, z_1 is unwound completely and no longer affects the price. The noise demand z_2 has a similar effect on the price dynamics.

2.1 Momentum and reversals

Next, we develop results that simultaneously allow for short-term reversals and longer-term momentum. Specifically, denote $E_\ell(\theta|s) \equiv \nu_{\theta_1}\kappa_s^{-1}s$ and $\kappa_{\theta|s} \equiv \nu_\theta - \nu_{\theta_1}^2\kappa_s^{-1}$ as respectively the mean and variance of θ conditional on s. Further, let $\kappa_{E_\ell(\theta|s)|F} \equiv (\nu_{\theta_1}^2/\kappa_s)(1-\kappa_s\kappa_F^{-1})$ be the variance of $E_\ell(\theta|s)$ conditional on F. We are able to prove the following proposition:

Proposition 2 *Let* $\nu_z \in [U_1, U_2]$*, where* U_1 *and* U_2 *are two positive numbers defined in Appendix A. We then have the following:*

(i) If μ *is sufficiently small, then long-run predictability* $\mathcal{L} > 0$*. Further,* $\mathcal{L}$ *decreases in* μ*.*

(ii) The parameter representing short-term predictability $\mathcal{S} < 0$*, and increases in* μ *if and only if* $2\kappa_{\theta|s} < \kappa_{E_\ell(\theta|s)|F}$*.*

[12] Indeed, observe that $a_2 = 0$ when $\mu = 0$.

(iii) As the magnitude of noise trading (ν_z) increases, for sufficiently small μ, momentum attenuates ($\mathcal{L}$ decreases) and short-term reversals strengthen ($\mathcal{S}$ becomes more negative).

Thus, in a finite range for the scale of noise trades, we obtain both short-term reversals (which obtain for high ν_z) as well as longer term momentum (which requires low ν_z). Within this range, increases in noise trading exacerbate short-run reversals and mitigate the momentum effect. If the noise trades unwind slowly (i.e., $\mu > 0$), then at Date 2 the demand effect of the unwound μz_1 causes a further deviation of P_2 from the fundamental; the expectation of the unwound μz_2's effect on P_3 causes a further deviation of P_2 from the fundamental. Both effects can decrease $\mathcal{L} = \text{Cov}(P_2 - P_0, P_4 - P_2)$.

It is intuitive that $\mathcal{S}$ becomes more negative as noise trades increase in scale, i.e., as ν_z increases. The effect of μ on $\mathcal{S}$, on the other hand, is more subtle. It may seem that allowing a greater proportion of noise traders to have long horizons (increasing μ) should attenuate short-horizon reversals (i.e., make $\mathcal{S}$ less negative). This does not follow from our model because investors speculate on the effects of noise trades on future prices. As information gets revealed in each period the risk premium required by active investors to absorb noise trades decreases, which contributes to short-horizon reversals even for high μ.[13] The extent of these risk premium reductions depends on the precision of the information signals. The proposition above shows that if $\kappa_{E_\ell(\theta|s)|F}$ is sufficiently high (i.e., if uninformed investors considerably underassess the content of the earnings announcement), then the effect of information revelation on the risk premium at the intermediate date is sufficiently weak that increasing μ attenuates short-term reversals, and vice versa.

In the remainder of this section, we assume that ν_z is in the range indicated by Proposition 2. We next show that the magnitude of the momentum effect is stronger if one skips a period between the portfolio holding and formation periods. This is because this skipping sidesteps the effect of the reversals. To show this, we define a parameter $\mathcal{L}^*$

$$\mathcal{L}^* \equiv \text{Cov}(P_2 - P_0, P_4 - P_3), \tag{6}$$

which represents the covariance where the return in the period between Dates 2 and 3 is skipped. We obtain the following result:

[13] For example, prices contemporaneously decrease when noise traders sell, but if the next period's signal is infinitely precise, prices reverse to their full information value regardless of whether noise traders want to hold stock beyond the date of the signal's release.

Proposition 3 *Skipping a period enhances the momentum effect, i.e., $\mathcal{L}^* > \mathcal{L}$; further, $\mathcal{L}^*$ decreases in ν_z.*

The above proposition is consistent with the finding that momentum profits are empirically enhanced by skipping one month between portfolio formation and holding; see, e.g., Korajczyk and Sadka (2004). To understand the result, we conduct a simple comparison between Equations (2) and (6), and obtain

$$\mathcal{L}^* = \mathcal{L} - \mathrm{Cov}(P_2 - P_0, P_3 - P_2);$$

thus, skipping effectively removes $\mathrm{Cov}(P_2 - P_0, P_3 - P_2)$ from $\mathcal{L}$. As part of the proof of Proposition 3, given in Appendix A, we show that because of the reversals induced by the noise demands z_1 and z_2, $\mathrm{Cov}(P_2 - P_0, P_3 - P_2) < 0$. By removing the effect of this covariance, skipping strengthens the momentum effect.

2.2 Transition from short-term reversals to momentum

Under reasonable conditions, short-term reversals can gradually transition to momentum as one conditions on progressively longer lags of returns to predict future returns. To formally show this, we define two parameters:

$$\begin{aligned} \mathcal{S}_{(2)} &= \frac{\mathrm{Cov}(P_1 - P_0, P_3 - P_2) + \mathrm{Cov}(P_2 - P_1, P_4 - P_3)}{2}, \\ \mathcal{S}_{(3)} &= \mathrm{Cov}(P_1 - P_0, P_4 - P_3). \end{aligned}$$

Compared to the short-term return predictability parameter $\mathcal{S}$, $\mathcal{S}_{(2)}$ represents lagging the return twice, and $\mathcal{S}_{(3)}$ represents lagging thrice. We obtain the following result:

Proposition 4 *(i) $\mathcal{S}_{(3)} > 0$.*

(ii) If $\mu = 0$, then $\mathcal{S}_{(2)} > 0$ and does not depend on the scale of noise trades ν_z.

(iii) If $\mu > 0$, then $\mathcal{S}_{(2)}$ decreases in ν_z. Specifically, as ν_z increases from zero, $\mathcal{S}_{(2)}$ is first positive and eventually turns negative.

Lagging the return by more than one period sidesteps the effect of the reversals induced by short-term noise traders. In this case, underreaction leads to price continuation; therefore, $\mathcal{S}_{(3)} > 0$, and provided that $\mu = 0$, $\mathcal{S}_{(2)} > 0$. If μ is high so noise trades have a longer impact on returns, then $\mathcal{S}_{(2)}$ may turn negative as the scale of noise trades, ν_z, increases. Since $\mathcal{S} < 0$, and $\mathcal{S}_{(3)} > 0$, Part (iii) of Proposition 4 implies that there exists a range of ν_z such that the level of return predictability at the second lag ($\mathcal{S}_{(2)}$) is bracketed by the levels at the first ($\mathcal{S}$) and third ($\mathcal{S}_{(3)}$) lags. It is evident that μ is crucial for this bracketing.

Note that the serial correlation in noise trades induced by a positive μ cannot generate unconditional momentum, as the effect of noise demands must eventually be reversed. This serial correlation, however, prolongs the horizon of the reversal, and thus helps explain how momentum can be offset by reversals during a transition period from reversals to momentum. Hence, noise traders, their horizons, and underreaction together help explain short-term reversals, longer-term momentum, and attenuated predictability in between.

2.3 Return predictability around earnings announcements

Denote $\text{Cov}_E \equiv \text{Cov}(P_2 - P_1, P_3 - P_2)$ to be the return autocovariance of the contiguous price changes around the earnings announcement. Further, let $\text{Cov}_{2E} \equiv \text{Cov}(P_2 - P_1, P_4 - P_3)$ denote the twice-lagged return autocovariance surrounding the earnings announcement. We obtain the analytical result below:

Proposition 5 *Provided that μ is sufficiently small, we have the following:*

(i) $\text{Cov}_E > \mathcal{S}$ *if* $\nu_\zeta/\kappa_f > \sqrt{3}\nu_\epsilon/\kappa_s$*. Further,* Cov_E *decreases in* ν_z*.*

(ii) $\text{Cov}_{2E} > 0$*.*

Since $\mathcal{S} < 0$, part (i) of the above proposition indicates that under the stated condition, short-term reversals attenuate around earnings announcements. The intuition is as follows. On the one hand, at Date 2, the price underreacts to earnings, and this underreaction is partially corrected at Date 3 when a more precise signal s is learned. This tends to raise Cov_E. On the other hand, the noise demand z_2 causes the price P_2 to deviate from the fundamental, and this deviation reverts across Dates 2 and 3. This tends to exert a downward pressure on Cov_E. Overall, Cov_E exceeds the short-term reversal parameter $\mathcal{S}$ under the condition in Part (i), which holds if the earnings announcement is sufficiently precise relative to the Date 3 signal (i.e., ν_ζ is high relative to ν_ϵ). This is because in this case, the continuation induced by prices' underreaction to earnings more than counteracts the reversals. Further, as previously shown (see the discussion following Proposition 4), lagging the return twice sidesteps the effect of the reversals induced by short-term noise traders, and this causes Cov_{2E} to be positive.

We observe here that when noise trades have a long-lasting impact on returns (i.e., noise traders hold their positions for more than two periods), lagging by two periods may not offset the effect of the reversals induced by noise traders. This case represents a variation from Proposition 5. While analytical solutions are not possible for this scenario, we present a numerical analysis in Section 3.4.

3 Numerical Analysis

This section performs numerical analysis for more general versions of our model. In this setting, all three noise demands, z_1, z_2, and z_3, are random, and informed investors have a non-trivial mass $0 < \lambda < 1$. To start, in Sections 3.1 we assume that noise trades at each date have a common variance $\nu_z > 0$, and calibrate the model to generate short-term reversals and longer-term momentum that roughly match the magnitudes observed in the U.S. data. Sections 3.2 and 3.3 then show that our central implications obtain in the neighborhood of the parameter set used for the calibration. Section 3.4 extends our model to a case where noise traders hold their positions to more than two periods, and to a scenario where the scale of noise trades can vary across time.

3.1 Short-term reversals and longer-term momentum: Calibrations

Figure 1 plots the short-term predictability parameter $\mathcal{S}$ as a function of the parameters representing the scale of the noise trades, ν_z, and the portion of noise trades that are long-term, μ. The other parameter values are $\lambda = 0.5$, $A = 2$, $\nu_\theta = 1$, $\nu_\xi = \nu_\epsilon = \nu_\zeta = 0.5$, and $\kappa = 2$.[14] It is notable that $\mathcal{S} < 0$ if ν_z (μ) is sufficiently high (low). Further, a greater scale of noise trades (i.e., higher ν_z) exacerbates short-term reversals (i.e., $\mathcal{S}$ becomes more negative). If noise trades unwind slowly (i.e., a higher μ), $\mathcal{S}$ tends to increase, though

[14] Our value for risk aversion, A, is same as that used in Leland (1992) and Holden and Subrahmanyam (2002). The value of unity for ν_θ is a normalization. The noisiness of the information signals, ξ, ϵ, and ζ, has a similar scale as that of the final payoff θ. We choose a neutral value for the mass of informed investors, λ, that is, 0.5 (the results are not particularly sensitive to the chosen value). Our value for the parameter κ is consistent with the range used by Odean (1998).

modestly.

Under plausible parameter values, our model generates short-term reversal magnitudes that are consistent with the empirical literature. To see this, we first note that the opposite of the short-term predictability parameter, $-\mathcal{S}$, can be interpreted as the average profit from trading on short-term reversals: that is, at each Date t ($t = \{1, 2, 3\}$), selling (buying) $|P_t - P_{t-1}|$ shares of the stock if $P_t - P_{t-1}$ is positive (negative), and holding this position until Date $t + 1$. The average long or short position (in shares) of this strategy at Dates 1, 2, and 3, denoted by $|Y_\mathcal{S}|$, is given by:

$$|Y_\mathcal{S}| = \frac{1}{3}\sum_{t=1}^{3} E(P_t - P_{t-1}|P_t - P_{t-1} > 0) = \sqrt{\frac{2}{9\pi}}\sum_{t=1}^{3}\text{std}(P_t - P_{t-1}),$$

where the last equality obtains because $P_t - P_{t-1}$ are all normally distributed with mean zero. Thus, the per-share payoff from trading against the short-term reversals is approximately $-\mathcal{S}/|Y_\mathcal{S}|$. We interpret each period (from one date to the next date) of our setting as representing 1.5 months (so that we can interpret the long-term autocovariance parameter as three-month momentum). Noting that we have normalized the payoff variance ν_θ to unity in our base parameter choices, we convert the per-share payoff to annual percentage units using an annual return standard deviation of 25% (Karolyi (2001)).[15] We plot the annualized return, $-\mathcal{S}/|Y_\mathcal{S}|$, in Figure 2. Depending on ν_z and μ, the average return from trading on short-term reversals can be negative or positive, and can be even higher than 10%. If, for example, $\mu = 0.2$ and $\nu_z = 0.15$, then it equals 9.4%. This is consistent with the number of 0.8% per month on the monthly reversal factor from Ken French's website.[16]

Next, Figure 3 plots the long-term predictability parameter $\mathcal{L}$ as a function of ν_z and μ. We see that momentum arises (i.e., $\mathcal{L} > 0$) when ν_z and μ are low. It is also notable that noise trades can mitigate or even reverse the momentum effect (i.e., $\mathcal{L}$ decreases and even becomes negative) when noise traders have a bigger scale (i.e., ν_z is high) and less of them unwind their positions quickly (i.e., μ is high); this result confirms the previous analysis in Proposition 2. As suggested by Chen and Hong (2002), the long-term predictability

[15] Since θ spans six months, the equivalent return standard deviation corresponding to ν_θ scales the annual value of 25% by $(1/2)^{0.5}$. This is then applied to the per share (1.5-month) payoff. The resulting number is then scaled up by eight to obtain annualized profits, so the scale factor applied to $-\mathcal{S}/|Y_\mathcal{S}|$ is $25\% \times (1/2)^{0.5} \times 8$.

[16] See http://mba.tuck.dartmouth.edu/pages/Faculty/ken.french/data library.html#Research.

parameter $\mathcal{L}$ can be interpreted as the average profit of a momentum strategy: that is, at Date 2, buying (selling) $|P_2 - P_0|$ shares of the stock if $P_2 - P_0$ is positive (negative), and holding this position until Date 4. The average long or short position (in shares) of this strategy, denoted by $|Y_{\mathcal{L}}|$, is given by:

$$|Y_{\mathcal{L}}| \;=\; E(P_2 - P_0 | P_2 - P_0 > 0) = \mathrm{std}(P_2 - P_0)\sqrt{2/\pi},$$

where the last equality obtains because $P_2 - P_0$ is normally distributed with mean zero. Thus, the per-share payoff from the momentum strategy is approximately $\mathcal{L}/|Y_{\mathcal{L}}|$. As in Figure 2, we convert the momentum payoff to annual percentage units using an annual return standard deviation of 25% (Karolyi (2001)).[17] We plot the annualized momentum payoff, $\mathcal{L}/|Y_{\mathcal{L}}|$, in Figure 4.[18] As we see, the average momentum return can range from being negative or zero to close to 10%. If, for example, $\mu = 0.2$ and $\nu_z = 0.15$, then it equals 3.3%. Given that our theoretical strategy does not go long-short in extreme deciles, but uses all stocks, this compares favorably with the 10% average (annual) return on international decile-based momentum in Rouwenhorst (1998) (Table III), and others.

An implication of Part (iii) of Proposition 2, as well as Figures 1 and 3 is that that for economies where ν_z is high, we would expect reversals, and for those with the opposite characteristic, we would expect momentum. Mediated by ν_z, we would expect momentum and reversal profits to be inversely related across countries. We test this implication in Section 4.

Effect of the underreaction parameter (κ): Figure 5 plots the momentum parameter $\mathcal{L}$ and the short term reversal parameter $\mathcal{S}$ as functions of κ, which represents the scale of underreaction. As we can see, as κ increases, momentum profits increase but reversal profits decline. The intuition is as follows. With an increase in κ, there is more underreaction, so momentum increases. However, an increase in κ means an underassessment of risk, which implies greater liquidity provision to noise traders and thus reduced reversals.

Effect of the precision of the public signal (ν_ϵ): Figure 6 plots the momentum parameter $\mathcal{L}$ and the short term reversal parameter $\mathcal{S}$ as a function of ν_ϵ, the parameter which represents the noisiness in the fundamental signal. As we can see, as ν_ϵ increases, momentum

[17] Note that in this case, the holding period is equivalent to three months. Thus, the scale factor of Footnote 15 is $25\% \times (1/2)^{0.5} \times 4$.

[18] The calibration of momentum follows similar exercises in other work such as Andrei and Cujean (2017).

profits decrease but reversal profits increase. The intuition is as follows. As ν_ϵ increases without bound, the fundamental signal becomes so noisy that active investors pay little attention to it. Without useful fundamental information, the underreaction to such information plays a negligible role in price formation, hence momentum is attenuated. On the other hand, with a greater ν_ϵ, there is more uncertainty about the asset's value, so the premium for bearing noise trader risk increases, thus increasing reversal profits.

Sections 3.2 and 3.3 to follow revisit some analytical results in Section 2 within the more general version of the model. Unless otherwise stated, the parameter values in these subsections are the same as those chosen for the calibration above.

3.2 Transition from reversals to momentum

We now confirm numerically in the general model that short-term reversals can be attenuated and gradually transition to momentum as one conditions on progressively longer lags of returns to predict future returns. Panel A of Figure 7 plots $\mathcal{S}$, $\mathcal{S}_{(2)}$ (which represents lagging returns twice), and $\mathcal{S}_{(3)}$ (which represents lagging returns thrice) as functions of the scale of noise trades, ν_z; we let $\mu = 0$ so that once a noise trade arises, all of it unwinds on the next date. Given a sufficiently high ν_z, $\mathcal{S} < 0$ because the effect of the reversals of the noise trades offsets the underreaction to information signals. Both $\mathcal{S}_{(2)}$ and $\mathcal{S}_{(3)}$ are positive because lagging by more than one period sidesteps the effect of the reversals induced by short-term noise traders.

In Panel B of Figure 7, we let $\mu = 0.2$ so that it takes two dates for a noise trade to unwind completely. It is notable that in this case, noise trades also cause $\mathcal{S}_{(2)}$ to decline. Specially, as ν_z increases, $\mathcal{S}_{(2)}$ is first positive and then turns indistinguishable from zero. This is because the effect of the continuing reversals of the noise trades offsets the underreaction to information signals. $\mathcal{S}_{(3)} = \mathrm{Cov}(P_1 - P_0, P_4 - P_3)$ remains positive because the effect of the noise traders does not last at the longer lag. Our analysis here confirms the discussion following Proposition 4 – as we can see, the degree of return predictability at the second lag is bracketed by the short-term reversals parameter and the return predictability beyond the second lag (i.e., $\mathcal{S} < \mathcal{S}_{(2)} < \mathcal{S}_{(3)}$).

3.3 Return predictability around earnings announcements

Next, we examine returns around the earnings announcement at Date 2 in the general model. Figure 8 plots $\text{Cov}(P_2 - P_1, P_3 - P_2) \equiv \text{Cov}_E$ as a function of the scale of noise trades, ν_z, and the portion of noise trades that are long-term, μ. Consistent with the previous analysis (see Proposition 5 and the ensuing discussion), Cov_E decreases in ν_z; it is negative unless ν_z is sufficiently low. The effect of noise trades is attenuated around the earnings announcement, but it still dominates that of underreaction.

Figure 9 simultaneously considers $\mathcal{S}$, Cov_E, and longer-lag return predictability as measured by $\text{Cov}(P_2 - P_1, P_4 - P_3) \equiv \text{Cov}_{2E}$. In Panel A, we set $\mu = 0$ so that all noise traders have short horizons. We see that Cov_E is greater than $\mathcal{S}$ provided that ν_z is sufficiently high. This result is consistent with the previous analysis (see Proposition 5 and the ensuing discussion). Cov_{2E} is positive regardless of the scale of ν_z; this is because lagging by more than one period sidesteps the effect of the reversals induced by short-term noise traders. Panel B of the figure considers the case of $\mu = 0.2$ (slow unwinding of noise trades). In this case, as noted in the discussion following Proposition 5, Cov_{2E} turns negative for large ν_z. This is because the effect of the continuing noise trade reversals more than offsets the underreaction to information signals. In either of Panels A and B, however, it can be seen that for sufficiently large ν_z, Cov_E is higher than $\mathcal{S}$ and lower than Cov_{2E} (as demonstrated in Proposition 5), so that reversals are attenuated around earnings announcements, and this attenuation becomes stronger as we move to predicting a return further away from the earnings date.

3.4 Other specifications of noise trading

In our setting to this point, each cohort of noise traders holds the initial position for at most two periods. We now modify this aspect of the model, and leave all other features unchanged. Specifically, a fraction μ_1 of the Date 1 noise traders now reverse their positions at Date 2, a fraction μ_2 at Date 3, and the remainder hold their positions till Date 4. Such a setting does not permit analytical solutions, but is readily solved numerically. The algebraic details are omitted for brevity.

In Figure 10 we plot the long-term predictability parameter $\mathcal{L}$ as a function of the new parameter μ_2. We set $\mu_1 = 0.5$, $\nu_z = 0.15$ and the other parameter values are the same as in earlier figures. We see from the figure that if μ_2 is high, we still obtain momentum. However, as it drops lower, $\mathcal{L}$ turns negative. This is because in this case, the reversal induced by longer-term noise trades more than counteracts the underreaction.

Next, in Figure 11, we plot the return predictability measures $\mathcal{S}$, $\mathcal{S}_{(2)}$, and $\mathcal{S}_{(3)}$ as a function of μ_2. These measures represent return predictability from lagging returns once, twice, and thrice, respectively, as in Section 2.2. We see that $\mathcal{S}_{(2)}$ remains bracketed by $\mathcal{S}$ and $\mathcal{S}_{(3)}$ throughout, supporting Proposition 4 and Figure 7. However $\mathcal{S}_{(3)}$ increases in μ_2. Thus, the lower the tendency of noise traders to hold their positions very long-term (the greater is μ_2), the stronger is the (positive) long-lag return predictability.

It is also of interest to explore the consequence of relaxing the assumption that all noise trades have a common variance ν_z. For illustrative purposes, we numerically solve a model variant where $\nu_{z1} = \nu_{z2}$ and the variance of the Date 3 noise trade, ν_{z3}, is a free parameter.[19] Again, the algebraic details are omitted for brevity. Figure 12 plots the reversal and momentum parameters, as well as momentum with a period skipped between the holding and formation periods, all as functions of the Date-3 noise trade, ν_{z_3}. The other parameter values are the same as in earlier figures. The central result is that higher ν_{z3} stimulates reversals and attenuates momentum. In our empirical work, we interpret ν_{z3} as the noise trade in the holding period month (in which profits to momentum and reversal strategies are measured). Under this interpretation, based on Figure 12, we expect momentum and reversal profits to vary inversely with each other in the time series.

4 Empirical Motivation and Analysis

In this section, we first discuss the extant evidence in the context of our model. We then develop and test novel empirical implications of our model.

4.1 Existing Evidence

As we mentioned in the introduction, the existing evidence of short-term reversals, longer-term momentum, and no predictability in between (Jegadeesh (1990)) is consistent with the implications of our model (Proposition 4). There also is recent international evidence that accords with our implications. For instance, though there is no evidence of momen-

[19] In unreported simulations, similar results obtain when ν_{z1} and ν_{z2} are allowed to vary independently.

tum in Chinese A shares (Docherty and Hurst (2018)), George, Hwang, and Li (2023)) find significant momentum excluding February, which they argue is special because of the behavior of retail investors around the Chinese New Year. Specifically, they find very strong return reversals in February, which is similar to the finding of January reversals in the U.S.[20] Given the observed February spike in losing stocks' turnover, they attribute the February reversals to retail investors' appetite for these losing stocks around the New Year. In a study of Singaporean stocks, Hameed, Ni, and Tan (2023) find no unconditional momentum, but significant momentum in high-priced, large cap stocks. They show that retail investor prefer low-priced, small cap stocks and institutional ownership is relatively high in high-priced, large cap stocks. These findings indicate that because retail investors are likely to be noise traders, the effect of their trades camouflage momentum in low-priced, small cap stocks. Medhat and Schmeling (2022) show that the largest U.S. stocks with the highest share turnover show evidence of short-horizon (monthly) momentum, instead of reversals. Since institutions are more likely to be active in larger stocks (Ferreira and Matos (2008)), this finding is consistent with retail investors being more active in the relatively smaller stocks, and the underreaction of informed institutions more than offsetting noise traders at monthly horizons within the larger stocks. These papers support our result in Proposition 2 that the noise trades of retail investors exacerbate reversals, whereas underreaction to fundamental information promotes momentum.

4.2 New Implications

Our analysis further suggests three previously-untested empirical implications. We provide these below, and reference the proposition and/or figure that suggest each:[21]

1. (Proposition 5 and Figure 8) Short-term reversals are attenuated in months that follow those with earnings announcements.

2. Momentum and reversal profits are inversely related across countries (Figures 1&3; Part (iii) of Proposition 2) and over time (Figure 12).[22]

[20]See George and Hwang (2004). China's tax-year is the same as the calendar year in the U.S., so the February seasonality is not associated with year-end tax effects.

[21]While Proposition 5 mentioned in the first implication below is derived for small μ, it is readily verified that it holds for the parameter values in Figure 8, which encompass the calibration in Section 3.1.

[22]This implication is based on the assumption that the principal source of variation across time and

3. (Part (iii) of Proposition 2) Greater noise trader imbalances in absolute terms, or greater variability in such imbalances, imply stronger short-term reversals. To test this implication, we use retail trade imbalances as a proxy for noise trades.[23]

We describe our data in the next subsection, and then provide empirical evidence consistent with these implications.

4.3 Data

The U.S. sample is comprised of all common stocks on CRSP with share codes 10 or 11, excluding stocks with prices below $1 or market capitalization below the 10% NYSE breakpoint at the end of month $t-1$, where t is the current month. We exclude small and low priced stocks to circumvent any market microstructure issues, and to ensure our results are not driven by microcaps (Fama and French (2008)). The sample period is January 1931 to December 2020.

The international sample is comprised of stocks with primary listing in one of the 22 countries that make up the MSCI Developed (ex-US) index and the 27 countries that make up the MSCI Emerging markets index.[24] Datastream is the data source for this sample. We restrict the sample to stocks that the indicator ISINID identifies as the primary security, and the primary exchange code (EXDSCD) is one of the stock exchanges in the countries within the sample. We exclude depository receipts (DRs), REITS, and preferred stocks.

countries is the volume (ν_z) of noise trades. There may also be variations in the amount of arbitrage capital. This would imply a positive relation between momentum and reversals (when arbitrage capital is high, momentum and reversals would both be weak). This can be viewed as a competing hypothesis. We also acknowledge that the level of overconfidence (skepticism) can vary across countries. As we will see in Section 4.6, however, we do control for individualism (the Chui, Titman, and Wei (2010) proxy for overconfidence).

[23] We use both the absolute retail imbalance during a month, and the monthly standard deviation (s.d.) of this imbalance, to measure the scale of noise trades. This is because under normality, the s.d. of the noise trade is proportional to its absolute value. Strictly speaking, the econometrician observes the unconditional expectation of the absolute noise trade (alternatively, the expected s.d.). The s.d.'s of noise trading during our four periods are the square roots of ν_z, $\nu_z(1+\mu^2)$, $\nu_z[1+(1-\mu)^2+\mu^2]$, and $\nu_z[1+(1-\mu)^2]$. The average of these quantities increases in ν_z, but its derivative with respect to μ has ambiguous sign. If μ is fairly stable, the bulk of the empirical variation in the absolute value and s.d. obtains from ν_z, and this parameter tends to make $\mathcal{S}$ more negative (from Proposition 2 and Figure 1).

[24] The developed market countries are Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Hong Kong, Ireland, Israel, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Singapore, Spain, Sweden, Switzerland, and the United Kingdom. The emerging market countries are Argentina, Brazil, Chile, China, Colombia, Czech Republic, Egypt, Greece, Hungary, India, Indonesia, Korea, Kuwait, Malaysia, Mexico, Pakistan, Peru, Philippines, Poland, Qatar, Russia, Saudi Arabia, South Africa, Taiwan, Thailand, Turkey, and the United Arab Emirates.

We compute monthly returns from Datastream, and use the data cleaning procedures suggested by Griffin, Kelly, and Nardari (2010), Hou, Karolyi, and Kho (2011), Ince and Porter (2006), Jacobs and Müller (2020), and Lee (2011).[25] We apply data filters in the following order. First, if the return in either of two consecutive months is greater than 300% and the cumulative return across those two months is less than 50% then we set returns in both months as missing. The rationale is that reversal of large returns implies a data error. Second, we discard all monthly returns greater than 200%. Also, if 90% or more of stocks in a country have zero returns during any month then we set the returns for all stocks in that country to missing values for that period. Finally, we winsorize the remaining returns in each country at the 0.1% and 99.9% levels.

Since our tests (and our model) involve simply relating monthly current returns to past returns, we do not adjust them for market or factor returns, nor do we include controls for other cross-sectional predictors. However, we have tried such adjustments based on the market factor and the global version of the five Fama and French (2015) factors, and including profitability (Novy-Marx (2013)) as well as asset growth (Cooper, Gulen, and Schill (2008)) as cross-sectional predictors; the results are unchanged.

4.4 Transition from reversals to momentum: An out-of-sample check

Jegadeesh (1990) (Table I) regresses returns on past lags in a manner similar to the following equation:

$$r_{i,t} = \rho_0 + \sum_{j=1}^{\tau} (\rho_j \times r_{i,t-j}) + \epsilon_{i,t}, \tag{7}$$

where $r_{i,t}$ is the return on stock i in month t, and the explanatory variables are monthly returns from $t-1$ through $t-\tau$. He shows that the first lag is negative and significant, the second lag is insignificant, while the remaining lags are positive and mostly significant. This transition accords with our model (viz. Proposition 4 and Figure 7). However, the data are for the U.S. and end in 1982. To investigate robustness, we first replicate the analysis (with $\tau = 12$)[26] over the updated 1931-2020 period, and then extend the analysis to an international context.

[25] We thank Amit Goyal for help with the data cleaning process.

[26] Jegadeesh (1990) also uses the 24th and 36th lag but these are not consistently significant in our sample and are omitted for brevity. While he adjusts returns for their long-run time-series means, he finds that this has little impact on the coefficients; hence we desist from doing this adjustment.

Panel A of Table 2 presents Fama-MacBeth regression estimates and t-statistics for U.S. stocks. We find that the estimate of ρ_1 is -0.045 with a t-statistic of -13.54, while ρ_2 is indistinguishable from zero. The slope coefficients ρ_3 through ρ_{12} are significantly positive. The sum of ρ_2 through ρ_{12} captures the cumulative momentum effect. Table 2 tabulates this sum, which is 0.1157 for the full sample period, and significantly exceeds zero. Further, we also present the χ^2-statistic for the null hypothesis that all 12 slope coefficients are jointly equal to zero; this is rejected at any conventional level of significance. The table also presents the regression estimates in three 30-year subperiods. The patterns in the slope coefficients during these subperiods are similar to those during the full sample period, although there is a diminution of the predictability in recent years.

Panel B of Table 2 presents a global out-of-sample test for the U.S. results in Panel A. Specifically, we report the Fama-MacBeth estimates of Regression (7) for the international sample over the January 1991 to December 2020 period. These results are strikingly similar to those for the U.S. For example, ρ_1 is significantly negative and it is the largest coefficient in magnitude. The coefficients ρ_2 through ρ_{12} are all positive, although ρ_2 and ρ_4 are not significantly different from zero. As with the U.S. data, the evidence that ρ_1 is significantly negative and ρ_3 is significantly positive, but that ρ_2 is insignificant, supports the return dynamics implied by our model. Note also that the sum of ρ_2 through ρ_{12}, which measures the cumulative momentum effect, is significantly positive. The absolute value of the monthly reversal coefficient and the sum of the momentum coefficients for non-U.S. stocks in Panel B are higher than those in the corresponding subperiod for U.S. stocks within Panel A. Further, the χ^2-statistic for Panel B is 96.3 and readily rejects the null of no predictability.

In sum, the out-of-sample findings confirm that the gradual transition from short-term reversals to momentum (which accords with Section 2.2 and 3.2 of our model) is robust.[27] Specifically, the finding that ρ_2 is indistinguishable from zero is consistent with Figure 7

[27] Ehsani and Linnainmaa (2022) show that factor momentum subsumes individual stock momentum (see also Kelly, Moskowitz, and Pruitt (2021) and Arnott, Kalesnik, and Linnainmaa (2023)). Falck, Rej, and Thesmar (2020) (p. 3) indicate that "factor momentum is "spanned" by stock momentum and factor exposure, except at one-month time scale." Further, the evidence from Ehsani and Linnainmaa (2022) is that factor momentum is strongest at the one-month horizon while we have one-month return reversals for individual stocks (Jegadeesh (1990)). As such, factor and stock momentum appear to haver non-overlapping elements, and we focus on stock-level momentum.

of Section 3.2 and supports the notion that the momentum effect due to underreaction just offsets reversals at the two-month horizon.[28] The fact that the sum of the slope coefficients ρ_3 through ρ_{12} is significantly positive accords with the idea that the effect of noise traders is offset by underreaction at horizons longer than two months.

4.5 Return predictability around earnings announcements

Our first untested implication in Section 4.1 indicates that reversals should be lower in the month following an earnings announcement. We test this implication separately for U.S. and non-U.S. stocks. We examine return dynamics following earnings announcements using the following regression:

$$r_{i,t} = \rho_0 + \sum_{j=1}^{12} (\rho_j \times r_{i,t-j}) + b \times EAD\ Dummy_{i,t-1} + \phi \times EAD\ Dummy_{i,t-1} \times r_{i,t-1} + \epsilon_{i,t}, \quad (8)$$

where $r_{i,t}$ is the return on stock i in month t and $EAD\ Dummy_{i,t-1}$ is an indicator variable that equals 1 if stock i announces earnings in month $t-1$. The coefficient ϕ captures the effect of the earnings announcement on short-term reversals. Since we expect a negative ρ_1 (from Table 2), a positive ϕ would suggest an attenuation of reversals following earnings announcements, and vice versa.

To estimate Equation (8), we obtain quarterly earnings announcement dates for U.S. stocks from Compustat and for international stocks from Worldscope. Because the availability of these dates in Compustat and Worldscope is more limited than returns data on CRSP or Datastream, we add the condition that a stock have at least one earnings announcement in the previous 12 months to be included in the sample for a particular month. Due to the requirement that earnings dates be available, the U.S. sample spans January 1972 to December 2020, while the international sample spans 1992 to 2020.

We fit Regression (8) using the Fama-Macbeth procedure. Panel A of Table 3 presents the estimates for U.S. stocks. The behavior of the lagged return coefficients is similar to that in Table 2. The coefficient ϕ, however, is significantly positive (with a t-statistic that exceeds seven), implying that monthly reversals attenuate when the previous month contains an earnings announcement. The magnitude of ϕ at 0.025 compares favorably

[28] Goyal and Wahal (2015) show that ρ_2 is negative and significant during 1927–1947 and 1969–1989, which is consistent with long-lasting noise trading during these subperiods.

to the size of the monthly reversal coefficient, which is -0.037. The coefficient estimate of ϕ suggests that reversals in the month following an earnings announcement are 68% (0.025/0.037) lower than their unconditional value. The sum $\rho_1 + \phi$ at -0.012 shows that over the one-month horizon, return reversals due to noise traders more than offset the short-horizon continuation in returns around the earnings announcement. This coefficient behavior qualitatively matches our model (viz. Proposition 5 and Figure 8).

Panel B of Table 3 presents the results for international stocks. The sample period for international stocks is from June 1992 to December 2020, which is a shorter period than our U.S. sample that starts in 1972. Again, the coefficient on the one-month lag is negative at -0.030 and strongly significant, while ϕ is positive and significant at 0.021 (t-statistic=3.99), showing that short-term reversals attenuate around earnings announcements. The magnitude of the attenuation is about 70% (0.021/0.030). The net effect of the first lag ($\rho_1 + \phi$) is -0.010, and the corresponding magnitude is comparable to that for the U.S.[29] Thus, international data yield the same qualitative result that reversals attenuate by more than two-thirds in months that follow earnings announcements. This evidence is strong support for the first implication in Section 4.1.

4.6 The relation between momentum and reversal profits

We now test our second implication in Section 4.1, that momentum and short-term reversal profits should be inversely related, across countries and in the time-series for each country. For these test, we form decile-based hedge portfolios based on past one-month and past two-to-twelve month returns within each country, and measure the returns on these portfolios the subsequent month. The reversal portfolio goes long the lowest return decile and vice versa, whereas the opposite is true for the momentum portfolio. Since we perform a cross-country analysis, we do not separate the U.S. from other countries in this subsection. We require countries to have 100 firms as of the previous December to ensure the reliability of portfolio returns.

[29]Table 4 uses the maximal time series available for earnings data in the U.S. (Panel A) and internationally (Panel B). In untabulated analyses, however, we replicate Panel A for the same period as Panel B (1992-2020), and find qualitatively similar results. Specifically, ρ_1 is negative and significant (although, consistent with Table 2, it is smaller in magnitude relative to that in Panel A), while ϕ is positive and significant.

4.6.1 Momentum and reversals across countries and time

We first explore the cross-country relation between momentum and short-term reversals. At the outset, we note that in untabulated findings, short-term reversal and momentum strategies yield positive average returns in 70% and 90% of the countries, respectively, and the bivariate cross-country correlation between these returns is a statistically significant -0.313. This latter finding supports a negative relation between momentum and reversals across countries. To confirm this relation, Table 4 runs a pooled cross-sectional, time-series regression of short-term reversal profits on momentum profits:

$$REV_{j,t} = \alpha_0 + \alpha_1 MOM_{j,t} + u_{j,t}, \tag{9}$$

where $REV_{j,t}$ and $MOM_{j,t}$ are short-term reversal and momentum profits in country j during month t. We present two versions of Regression (9), with and without month fixed effects. As can be seen, in each case, the slope is strongly negative, with an absolute t-statistic exceeding six. In terms of magnitude, from Table 4, a one standard deviation increase in momentum implies a 0.19 standard deviation decrease in reversals, which is substantial.[30] We acknowledge that the choice of dependent and independent variable is arbitrary in Table 4; however, reversing the choice leads to an identical conclusion that momentum and short-term reversal profits are negatively related across countries. We build on this finding by linking these profits to an exogenous mediating variable within Table 6 to follow.

We now examine whether momentum and reversal profits are negatively related in the time-series. We would expect such a relation from Figure 12, as discussed in Section 3.4. In Table 5, we present the time-series correlations between contemporaneous momentum and short-term reversal profits, that is, $\mathrm{Corr}(REV_{j,t}, MOM_{j,t})$, for each country j in our sample. As we can see, this correlation is negative for 90% of the countries. Further, the correlation exceeds 10% in absolute terms for each of these negative cases. Finally, the average correlation is highly significant at -0.216 (with a median of -0.251). This evidence also is consistent with our second implication that momentum and short-term reversal profits should vary inversely with each other.

[30] This conclusion is based on the coefficient of -0.166 from the regression which includes month fixed effects. The standard deviation of reversal profits is 89% of that of momentum profits.

4.6.2 Cross-country differences that explain return predictability

This section explores cross-country differences that might generate the observed correlations in momentum and reversal profits. Our model suggests that these differences may come from institutional and cultural differences across countries that influence the behavior of either noise traders or active informed traders. It would be natural to directly consider cross-country differences in noise trading and institutional ownership, however, we are not aware of country-by-country data on retail trading (or even holdings) and institutional ownership data tend to be very noisy (Ferreira and Matos (2008)). Moreover, in an international setting it is difficult to classify large blockholdings and insider ownership (Becht and Röell (1999)) within the context of our model.

Cultural differences may provide a more promising path for identifying relevant cross-country differences. For example, the finance literature has explored two dimensions of culture, identified by Hofstede (2001) that appear to influence stock return patterns. Thus, Chui, Titman, and Wei (2010) propose that the individualism trait of Hofstede (2001) leads to excessive overconfidence, which is then linked to momentum. In addition, Nguyen and Truong (2013) consider the Hofstede (2001) uncertainty avoidance trait and find that the information content of stock prices is higher in countries with low uncertainty avoidance, supporting the view that there may be more focus on fundamentals in such countries.[31] Since momentum arises from trading on long-term fundamentals while reversals are caused by informationless noise trades within our setting, we expect low uncertainty avoidance to lead to more momentum and weaker reversals.

Details of the data collection for the country-specific cultural traits are described in Chui, Titman, and Wei (2010) and Hofstede (2001).[32] Summary statistics for the attributes of individualism (IDV) and uncertainty avoidance (UAI) appear in Panel A of Table 6. The means and medians for each attributes are relatively close to each other, indicating little skewness. The correlation between individualism and uncertainty avoidance is moderately negative at -0.167.

[31] As Nguyen and Truong (2013) indicate, uncertainty avoidance refers to "the extent that people of a culture feel threatened by uncertain or unknown situations and the extent that people try to minimize such uncertainty" (Hofstede (1984)), and this might reduce a focus on uncertain long-run fundamentals and increase noise trading.

[32] These attributes are discussed and available at https://tinyurl.com/333bxra2. The data are available for 66 countries.

We first conduct an exploratory exercise relating UAI and IDV to momentum and short-term reversals. Specifically, Panel B of Table 6 presents average momentum and short-term reversal profits across countries (in percentages per month) split by median values of UAI and IDV. Momentum profits are higher (1.65%) in countries with above-median IDV than those with below-median IDV (0.961%), which confirms Chui, Titman, and Wei (2010). We also see, however, that short-term reversal profits are three times higher (0.609%) in countries with above-median UAI than in those with below-median UAI (0.233%). Momentum and reversal profits are higher in low UAI and high IDV countries, respectively, relative to countries with the opposite characteristics, but the magnitude of these differences is less dramatic.

To investigate the patterns in Panel B via regression, Panel C of Table 6 presents the results of the following pooled cross-sectional, time-series regressions that relate reversals and momentum to IDV and UAI:[33]

$$PRED_{j,t} = \alpha_0 + \alpha_1 CULTURE_j + u_{j,t}, \tag{10}$$

where $PRED_{j,t}$ is the short-term reversal or momentum profit in country j during month t, and $CULTURE_j$ is a matrix that consists of either one or two cultural attributes (IDV or UAI). The results from estimating Regression (10) again confirm the result of Chui, Titman, and Wei (2010) that momentum is positively related to IDV. However, IDV is not related to reversals. Further, UAI is significantly related to both reversals and momentum; it implies stronger short-term reversals and weaker momentum. The effect of UAI on reversals has a t-statistic that exceeds five in absolute terms.[34] When both IDV and UAI are included as explanatory variables in the same regression, UAI continues to be significant at the 1% level for reversals, and at the 10% level for momentum. In terms of magnitudes, considering the bivariate regressions, a one standard deviation increase in uncertainty avoidance implies a 30 basis point increase in short-term reversals, and an 18

[33]We include month x developed market status fixed effects, but the results are not sensitive to this inclusion.

[34]We also control for other measures of investor sophistication which might be inversely related to the level of noise trading, such as per capita GDP (which might be related to education in general, and financial education specifically), and a direct measure of financial literacy (obtained from Klapper, Lusardi, and Van Oudheusden (2015)). We also include a measure of volatility, which is calculated as a rolling standard deviation of monthly returns over the immediately preceding 24 months. These variables do not have an impact on the role of UAI.

basis point decrease in momentum. The results support the view that lower uncertainty avoidance implies stronger momentum and weaker short-term reversals.

4.7 Retail order imbalance and short-term reversals

We use the trades of retail investors to test the third and last implication in Section 4.1, that reversals are stronger when noise trader imbalances are more variable or higher in absolute terms. In doing so, we do not mean to imply that *all* retail trades are noise trades; just that such trades are more likely to emanate from individual investors (see Barber, Lee, Liu, and Odean (2008)). We estimate the net trade imbalance of retail investors using the method suggested by Boehmer, Jones, Zhang, and Zhang (2021) to isolate retail trades.[35] Specifically, in month t, we compute

$$Net\ Retail\ Buy_{i,t} = \frac{Retail\ Buy_{i,t} - Retail\ Sell_{i,t}}{Retail\ Buy_{i,t} + Retail\ Sell_{i,t}},$$

where $Retail\ Buy_{i,t}$ and $Retail\ Sell_{i,t}$ are the number of shares of stock i bought and sold by small investors in month t, as reported in TAQ. The sample consists of all common stocks with available data for the TAQ variables $BuyVol\_Retail$ and $SellVol\_Retail$ on a daily basis, which are the imputed levels of retail buy and sell volume, respectively. These variables are each summed to the monthly level for each stock. The monthly versions of these variables then form the $Retail\ Buy$ and $Retail\ Sell$ variables in the computation of $Net\ Retail\ Buy$ above.

We subtract the cross-sectional mean of $Net\ Retail\ Buy_{i,t}$, and then divide by the cross-sectional standard deviation to compute a scaled measure of retail order imbalance, which we denote as $Retail\ OIB_{i,t}$, and we use its absolute value $|Retail\ OIB|_{i,t}$ as a proxy for the magnitude of noise trading. We then perform the following Fama-MacBeth regression:

$$r_{i,t} = \rho_0 + \rho_1 \times r_{i,t-1} + \rho_2 \times |Retail\ OIB|_{i,t-1} + \rho_3 \times r_{i,t-1} \times |Retail\ OIB|_{i,t-1} + \epsilon_{i,t}, \quad (11)$$

where we interact lagged returns with the absolute retail imbalance. Our sample period and cross-section is limited by the availability of retail imbalances to November 2006 –

[35] In a recent paper Barber et al. (2021) indicates that the Boehmer, Jones, Zhang, and Zhang (2021) procedure faces challenges when bid-ask spreads are high. Any errors in classification, however, should attenuate the measured coefficients.

December 2021, and to U.S. stocks.[36]

Column (1) of Table 7 includes only the one-month lagged return, while column (2) includes all the variables in Regression (11). As we see, unconditionally, monthly reversals are significant with a t-statistic of -2.83, even for our smaller and more recent sample period over which retail trades are available. In column (2), however, we see the following pattern. First, the coefficient of the lagged return more than halves in magnitude when the absolute retail imbalance terms are included, and becomes insignificant. We also see that the coefficient on the interaction term, ρ_3, is strongly significant and negative with a t-statistic of -3.18. This indicates that absolute retail imbalances do exacerbate monthly reversals, which accords with our model.[37] Both the baseline coefficient on lagged return and the interaction coefficient of lagged return with retail imbalance have a similar (absolute) magnitude of about 0.02, and the full-sample standard deviation of $|Retail\ OIB|$ is 0.67. This means that a one-standard-deviation increase in retail imbalance has an impact on monthly reversals that is about two-thirds of the baseline effect.

In our theoretical model, the absolute level of noise trades is proportional to the standard deviation of noise trades because we assume normality. This correspondence does not have to be exact empirically, because of possible departures from normality and imperfect trade signing. As a robustness check, therefore, in column (3) of Table 7 we replace $|Retail\ OIB|_{i,t-1}$ with $\sigma(Retail\ OIB)_{i,t-1}$, the monthly standard deviation of daily order flows from retail investors (which is also cross-sectionally standardized). As can be seen, the interaction of $\sigma(Retail\ OIB)_{i,t-1}$ with lagged returns continues to retain a negative sign and attains strong significance. Overall, the results confirm the role of retail demand (which is more likely to represent noise trades) as a key source of reversals.

[36]This recent U.S. period, which includes the global financial crisis, does not yield predictability beyond the first return lag (Daniel and Moskowitz (2016), McLean and Pontiff (2016), and Bhattacharya, Li, and Sonaer (2017)), nor any evidence that retail trades help explain return lags beyond the first, hence we refrain from including longer return lags in Equation (11).

[37]Barber, Huang, Odean, and Schwarz (2022) show that net (signed) order flows of retail investors at the discount brokerage RobinHood are accompanied by price movements in the opposite direction at 20-day horizons; this is consistent with our results at monthly horizons.

5 Discussion

We now discuss the consequences of relaxing the model's assumptions, and propose that alternative approaches might lead to additional insights.

5.1 Liquidity Provision Delays

In our model, all active investors are potential liquidity providers. Specifically, a shock to noise trades influences the positions of all active investors. In reality, many active investors have relatively long horizons and are unable to allocate their attention to all stocks at all times. A potential extension of our analysis would allow some active investors to be present in the market every period (as in our setting), but some to be inattentive to opportunities for liquidity provision. In this setting the investors who are continuously present would provide the initial liquidity; however, as other active investors enter the market, the inventory premium would decline. We believe that such an extension would explain the attenuation in the one-month reversal in recent years, corresponding with the participation of quant traders, with much lower costs of attention, and faster responses to order flows of noise trades.[38]

5.2 Longer-Term Reversals

Our model does not focus on long-term reversals over horizons of three years or more (De Bondt and Thaler (1985)). However, other models do address this phenomenon. For example, in Daniel, Hirshleifer, and Subrahmanyam (1998) long-term reversals arise because investors are overconfident about the precision of their own information signals and thus overreact to such signals. Such reversals also arise in Hong and Stein (1999) and Bordalo, Gennaioli, Ma, and Shleifer (2020) because trend-chasers cause overreaction of prices to past news. Assuming that people overestimate the precision of their own information, or naïvely extrapolate future returns from past long-term returns could help explain short-term reversals, intermediate-term momentum, and longer-term reversals, but would result in an exceedingly complex and perhaps less intuitive model.

[38] See Nagel (2012) and Cheng, Hameed, Subrahmanyam, and Titman (2017) for evidence that reversals decline with rises in liquidity provision and institutional capital available for arbitrage.

5.3 Acquisition of Noise Information

Our model raises the possibility that it may be profitable to acquire noise-related signals, particularly when such trades are positively autocorrelated (as in meme-stock related episodes). Trading in advance of anticipated noise trades would lead to additional rents for informed traders, and a model that endogenizes acquisition of signals about autocorrelated noise trades would be interesting. While we believe the thrust of our reversal/momentum results would remain unchanged in an extension, how information about dynamic noise trading affects the incentives to acquire and trade on fundamental information would form an interesting investigation.[39]

6 Conclusion

Returns gradually transition from reversals at short lags, to weak or no predictability at intermediate lags, and to momentum at longer lags. We develop an integrated framework that explains these features of equity markets and we test several new implications of the model. Our setting includes informed traders, "liquidity" or "noise" traders, as well as uninformed investors who underreact to information they do not themselves produce. In our model, underreaction to long-term fundamentals gradually offsets noise-trade-induced-reversals as the return lag length increases, provided noise trading is not too extreme, and if a sufficiently large proportion of noise traders liquidate positions quickly.

We test new predictions of our model. First, our analysis implies that reversals should attenuate following earnings announcements, because underreaction to earnings should counteract noise-trader-induced reversals. We find strong support for this attenuation. We also predict a negative relation between momentum and short-term reversal profits across countries and within each country across time. The data confirm this novel implication. We next propose that cross-country variations in momentum and reversal profits may in part be generated by cultural differences (Hofstede (2001)). We provide evidence that such variations may be due to differences in the trait of uncertainty avoidance. Specifically, countries with lower uncertainty avoidance have weaker reversals and stronger momentum. Our explanation, consistent with Nguyen and Truong (2013), is that

[39] Farboodi and Veldkamp (2020) and Yang and Zhu (2020) present models where some investors have information about noise trades, but such trades are not positively autocorrelated.

less uncertainty avoidance might imply more focus on uncertain long-term fundamentals and less noise trading, and, as suggested by our model, imply strengthened momentum and attenuated short-term reversals. Finally, our model implies that short-term reversals are stronger when noise traders' order flows are either more variable or higher in absolute terms. Using estimated retail order flows, we find support for this implication as well.

Our analysis suggests a future direction for empirical work that relates to how investor clienteles and information flows might influence return patterns across markets. In our model, momentum is caused by traders' reaction to new information. This observation indicates that the frequency of information releases, as well as whether noise traders or active investors dominate the market, should influence the prevalence of reversals versus momentum, as well as the horizons over which these phenomena occur. Along these lines, Medhat and Schmeling (2022) show that large stocks with high turnover show signs of momentum in the U.S., suggesting the dominance of active institutions and higher frequency of information releases in the large stock/high turnover segment. Going beyond this finding, identifying international proxies for noise trading and markets' information environments are likely to be challenging exercises, but warrant future research.

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Figure 1: Short-term reversals

This graph plots the short-term return predictability parameter $\mathcal{S}$ as a function of the parameters representing the scale of noise trades, ν_z, and the portion of noise trades that are long-term, μ. The other parameter values are $\lambda = 0.5$, $A = 2$, $\nu_\theta = 1$, $\nu_\xi = \nu_\epsilon = \nu_\zeta = 0.5$, and $\kappa = 2$.

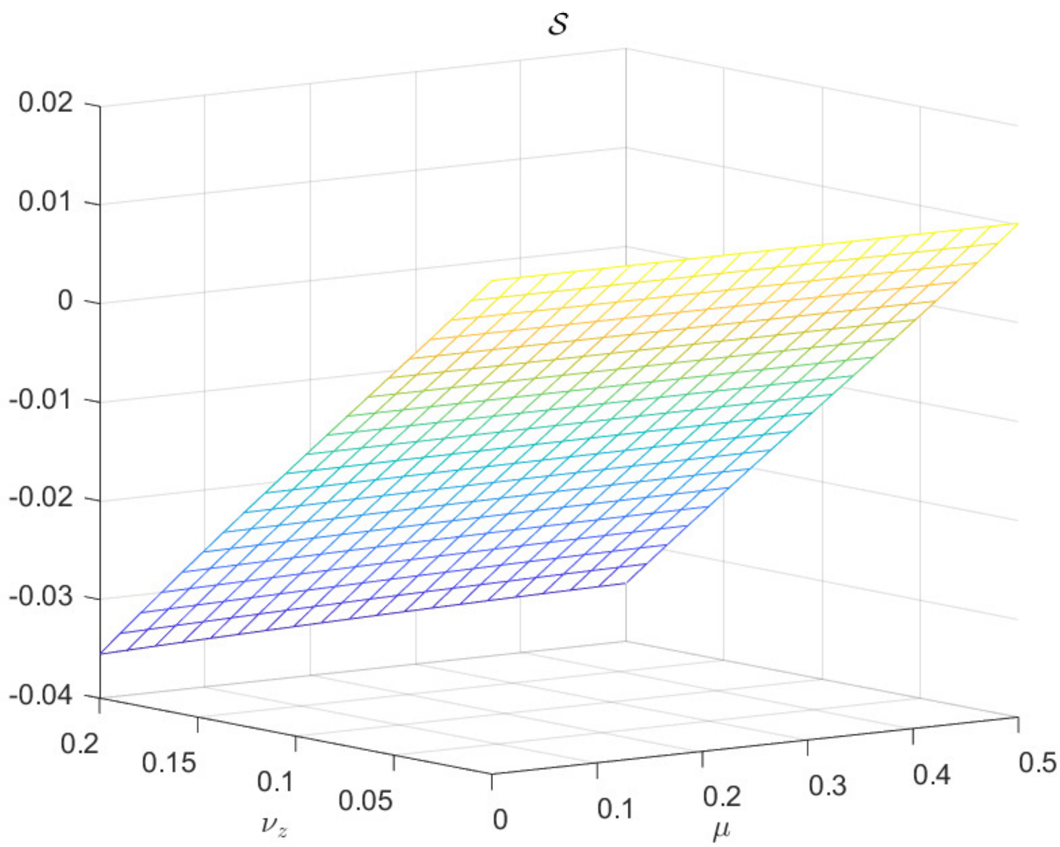


Figure 2: Trading on short-term reversals: Annual return

This graph plots the calibrated annualized returns of trading against short-term reversals, $-\mathcal{S}/|Y_{\mathcal{S}}|$, as a function of the parameters representing the scale of noise trades, ν_z, and the portion of noise trades that are long-term, μ. The other parameter values are $\lambda = 0.5$, $A = 2$, $\nu_\theta = 1$, $\nu_\xi = \nu_\epsilon = \nu_\zeta = 0.5$, and $\kappa = 2$.

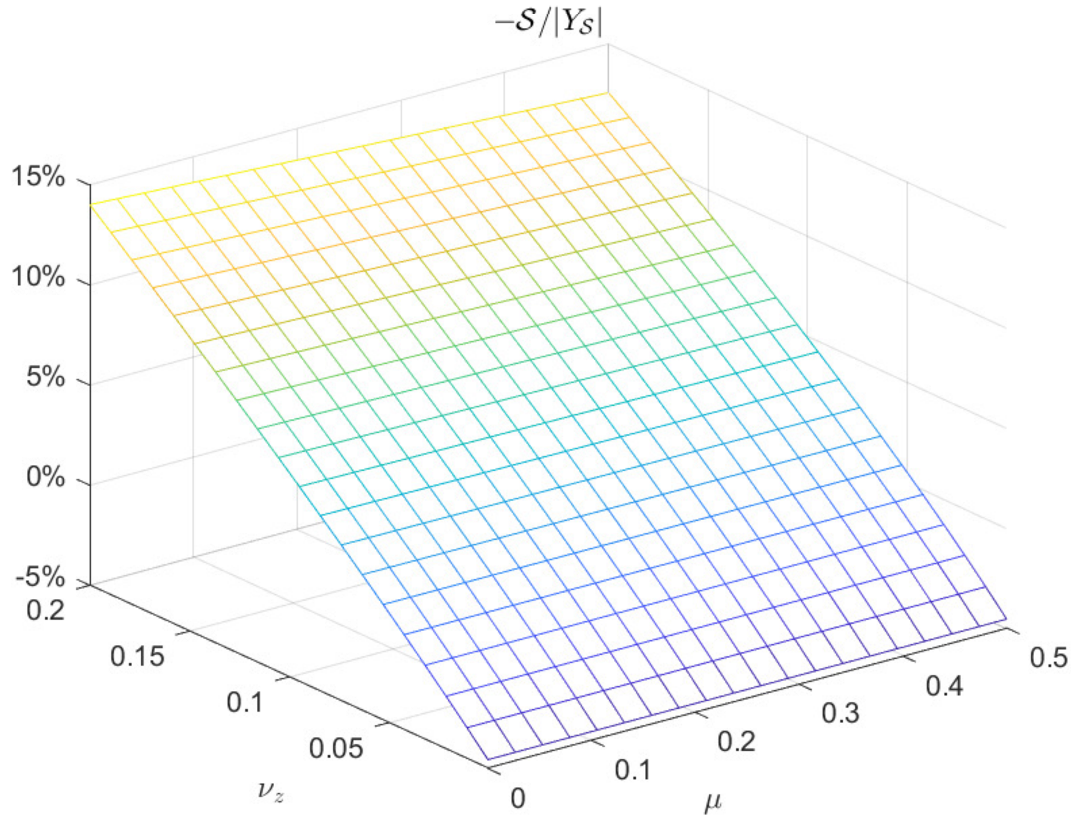


Figure 3: Momentum

This graph plots the long-term return predictability parameter $\mathcal{L}$ as a function of the parameters representing the scale of noise trades, ν_z, and the portion of noise trades that are long-term, μ. The other parameter values are $\lambda = 0.5$, $A = 2$, $\nu_\theta = 1$, $\nu_\xi = \nu_\epsilon = \nu_\zeta = 0.5$, and $\kappa = 2$.

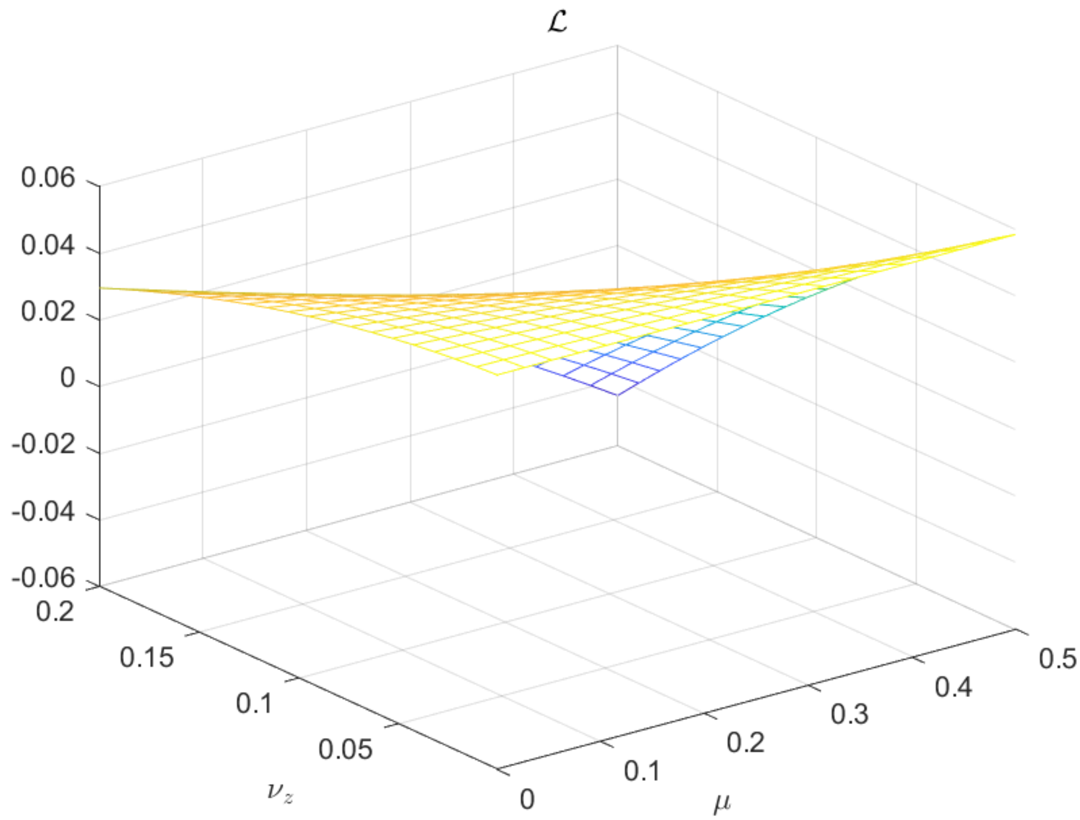


Figure 4: Annualized momentum return

This graph plots the calibrated annualized returns of the momentum parameter, $\mathcal{L}/|Y_{\mathcal{L}}|$, as a function of the parameter representing the scale of noise trades, ν_z, and the portion of noise trades that are long-term, μ. The other parameter values are $\lambda = 0.5$, $A = 2$, $\nu_\theta = 1$, $\nu_\xi = \nu_\epsilon = \nu_\zeta = 0.5$, and $\kappa = 2$.

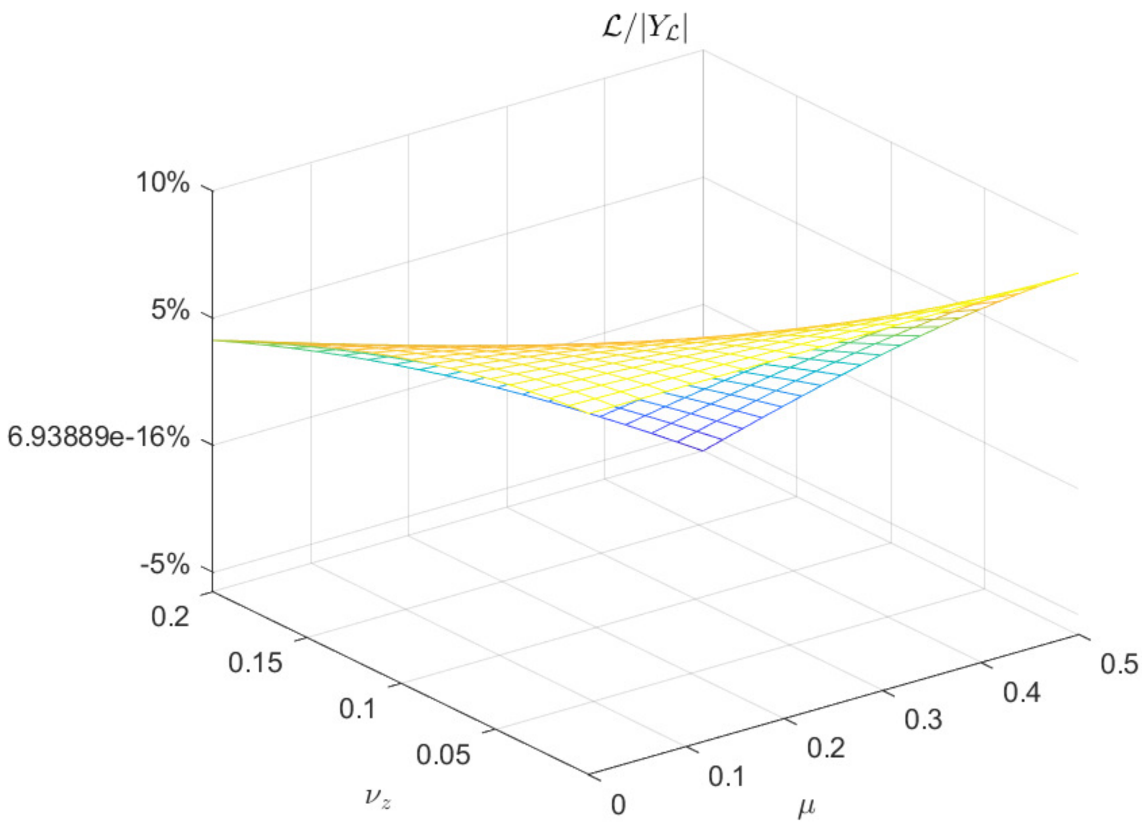


Figure 5: Momentum and reversals as functions of the scale of underreaction κ

This graph plots the long- and short-term return predictability parameters ($\mathcal{L}$ and $\mathcal{S}$, respectively) as functions of the parameter representing the scale of underreaction κ. The other parameter values are $\lambda = 0.5$, $\mu = 0.2$, $A = 2$, $\nu_\theta = 1$, $\nu_\xi = \nu_\epsilon = \nu_\zeta = 0.5$, and $\nu_z = 0.15$.

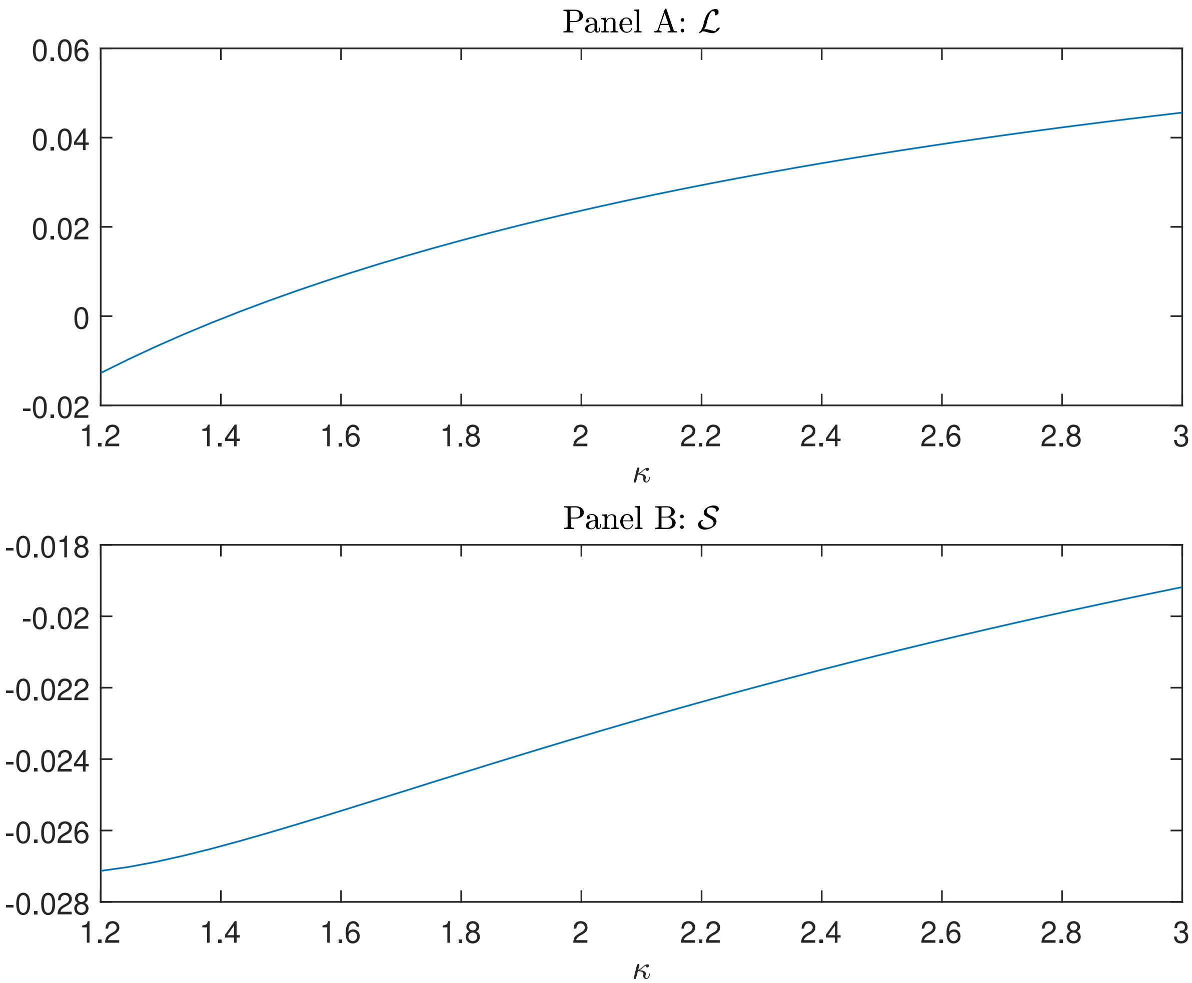


Figure 6: Momentum and reversals versus the noisiness of the fundamental signal ν_ϵ
This graph plots the long- and short-term return predictability parameters ($\mathcal{L}$ and $\mathcal{S}$, respectively) as functions of the parameter representing the noisiness of the public signal F, ν_ϵ. The other parameter values are $\lambda = 0.5$, $\mu = 0.2$, $A = 2$, $\nu_\theta = 1$, $\nu_\xi = \nu_\zeta = 0.5$, $\kappa = 2$, and $\nu_z = 0.15$.

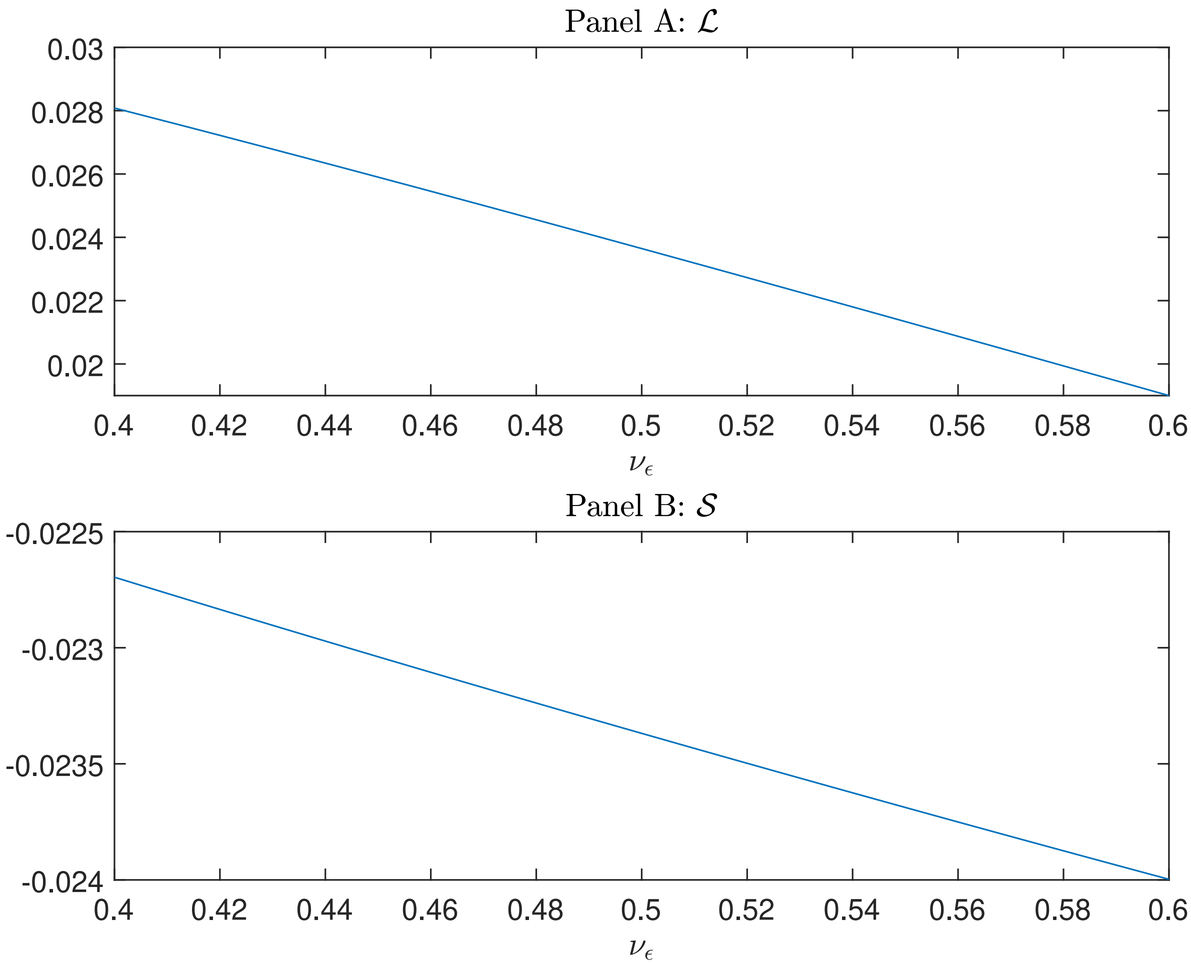
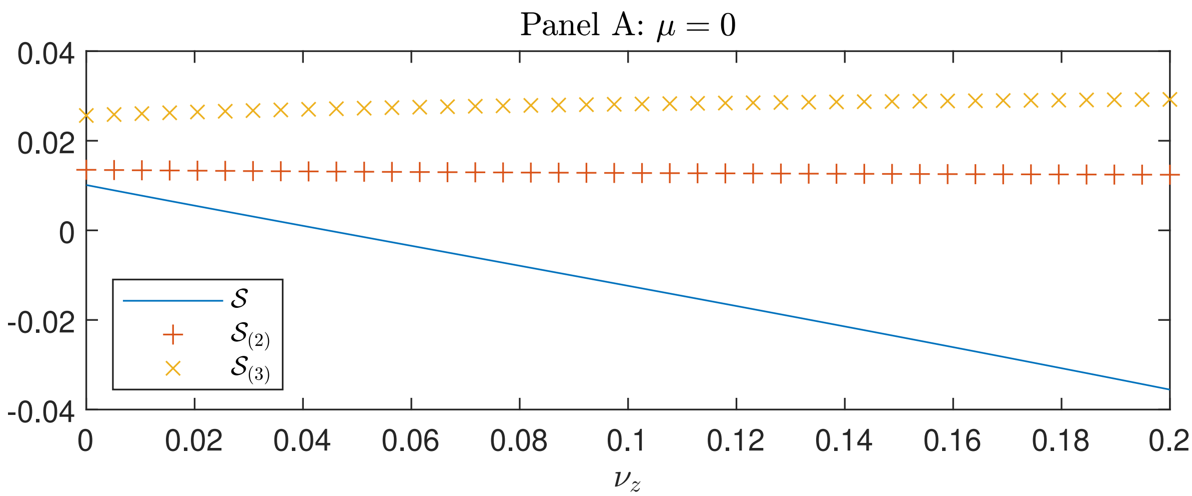


Figure 7: Return predictability conditional on longer return lags

This graph plots $\mathcal{S}$ (the parameter representing short-term return predictability), $\mathcal{S}_{(2)}$ (which represents lagging the return twice), and $\mathcal{S}_{(3)}$ (which represents lagging the return thrice) as functions of the scale of noise trades, ν_z. We let $\mu = 0$ in Panel A, and $\mu = 0.2$ in Panel B. The other parameter values are $\lambda = 0.5$, $A = 2$, $\nu_\theta = 1$, $\nu_\xi = \nu_\epsilon = \nu_\zeta = 0.5$, and $\kappa = 2$.



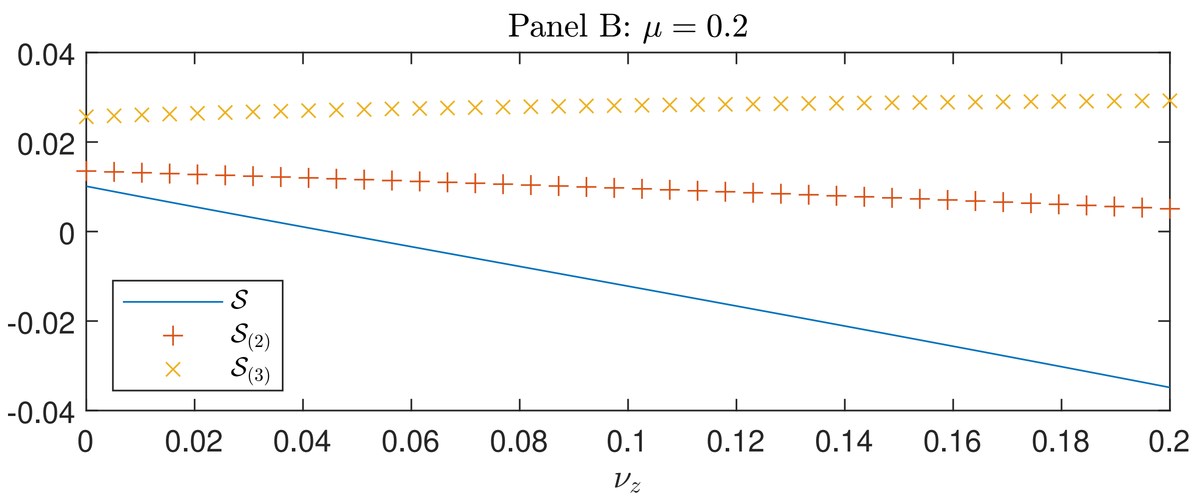


Figure 8: Return predictability conditional on the earnings announcement date

This graph plots $\mathrm{Cov}_E \equiv \mathrm{Cov}(P_2 - P_1, P_3 - P_2)$ as a function of the parameters representing the scale of noise trades, ν_z, and the portion of noise trades that are long-term, μ. The other parameter values are $\lambda = 0.5$, $A = 2$, $\nu_\theta = 1$, $\nu_\xi = \nu_\epsilon = \nu_\zeta = 0.5$, and $\kappa = 2$.

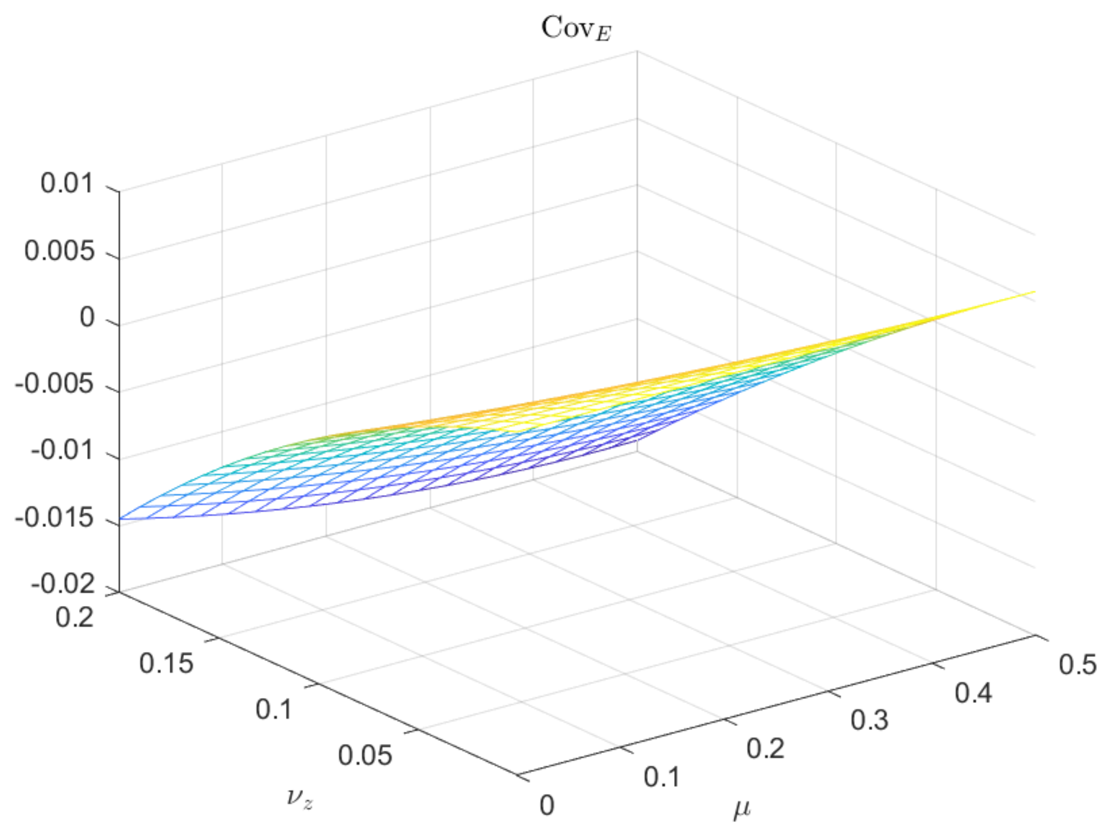


Figure 9: Longer-lag return predictability conditional on the earnings date

This graph plots the short-term return predictability parameter $\mathcal{S}$, $\text{Cov}_E \equiv \text{Cov}(P_2 - P_1, P_3 - P_2)$, and $\text{Cov}_{2E} \equiv \text{Cov}(P_2 - P_1, P_4 - P_3)$ as functions of the parameter representing the scale of noise trades, ν_z. We let $\mu = 0$ in Panel A, and $\mu = 0.2$ in Panel B. The other parameter values are $\lambda = 0.5$, $A = 2$, $\nu_\theta = 1$, $\nu_\xi = \nu_\epsilon = \nu_\zeta = 0.5$, and $\kappa = 2$.

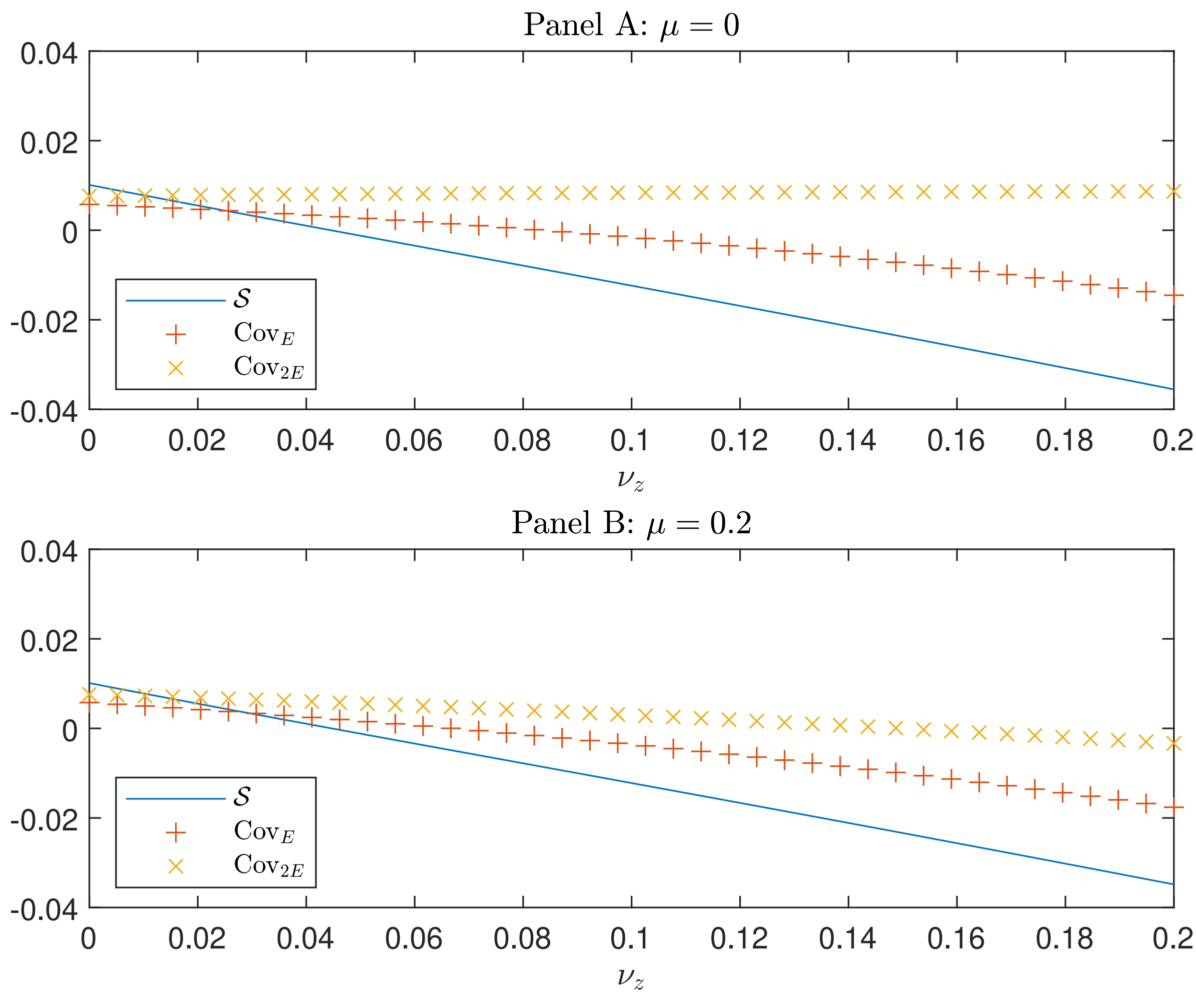


Figure 10: Momentum when noise traders have very long horizons

This graph plots the long-term return predictability parameter $\mathcal{L}$ as a function of μ_2, the proportion of Date 1 noise traders that reverse their positions at Date 3. We let $\mu_1 = 0.5$, and $\nu_z = 0.15$. The other parameter values are $\lambda = 0.5$, $A = 2$, $\nu_\theta = 1$, $\nu_\xi = \nu_\epsilon = \nu_\zeta = 0.5$, and $\kappa = 2$.

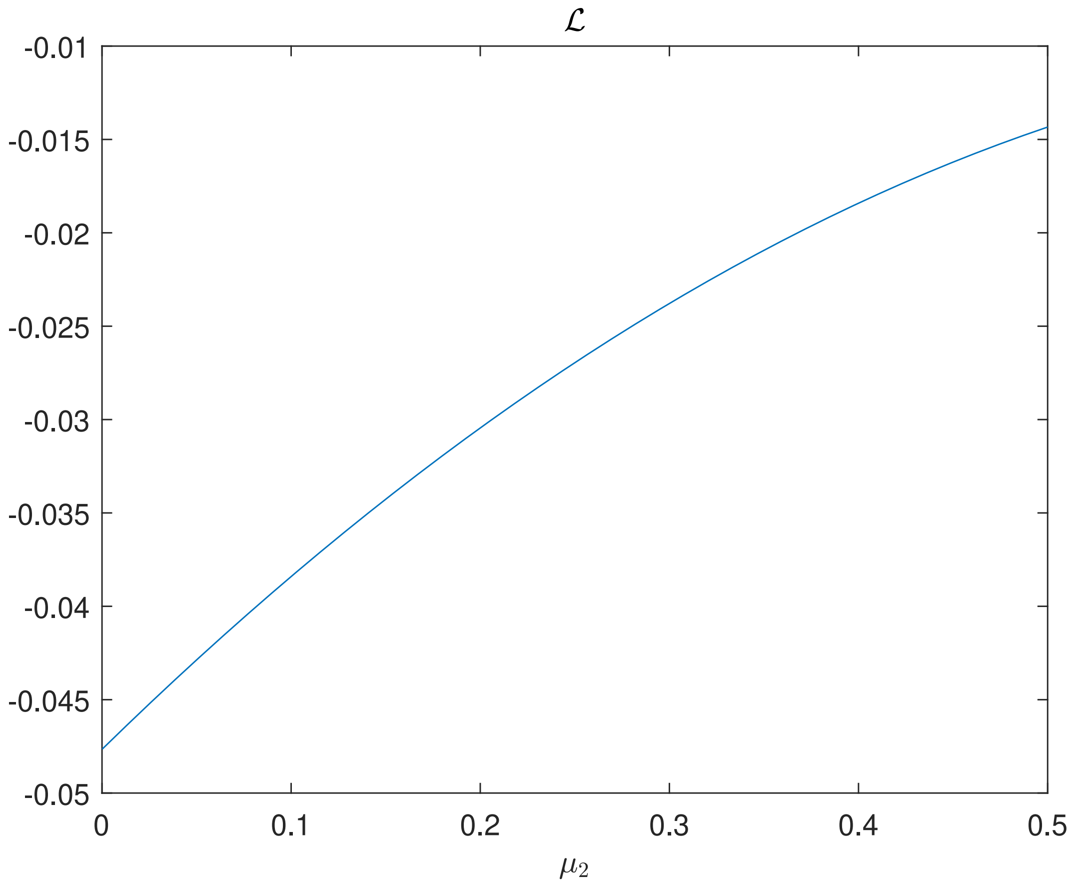


Figure 11: Long-lag return predictability when noise traders have very long horizons

This graph plots the short-term return predictability parameter $\mathcal{S}$, $\mathcal{S}_{(2)}$ (which represents lagging the return twice), and $\mathcal{S}_{(3)}$ (which represents lagging the return thrice) as functions of μ_2, the proportion of Date 1 noise traders that reverse their positions at Date 3. We let $\mu_1 = 0.5$, and $\nu_z = 0.15$. The other parameter values are $\lambda = 0.5$, $A = 2$, $\nu_\theta = 1$, $\nu_\xi = \nu_\epsilon = \nu_\zeta = 0.5$, and $\kappa = 2$.

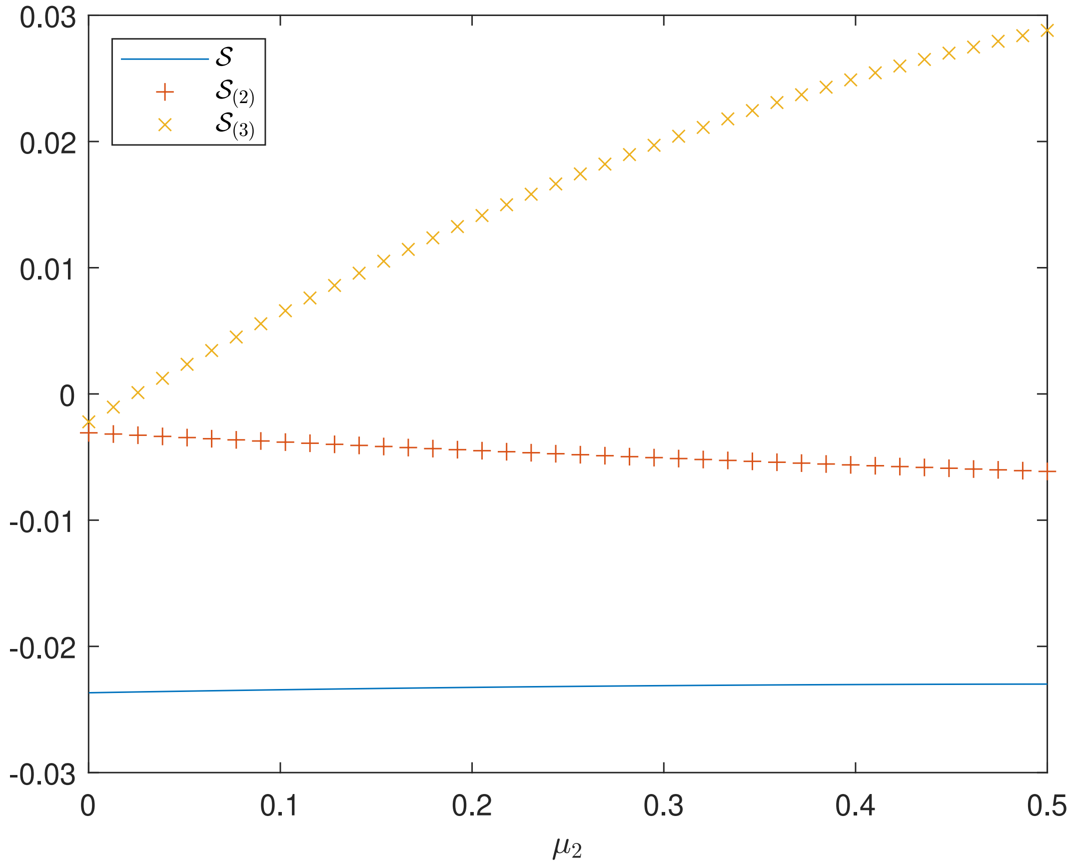


Figure 12: Reversals and momentum as functions of the Date 3 noise trade ν_{z_3}

This graph plots the short- and long-term return predictability parameters ($\mathcal{S}$ and $\mathcal{L}$, respectively), and the long-term return predictability parameter with a period skipped between past and future returns ($\mathcal{L}^*$), as functions of the parameter representing the scale of Date-3 noise trade, ν_{z_3}. The other parameter values are $\lambda = 0.5$, $\mu = 0.2$, $A = 2$, $\nu_\theta = 1$, $\nu_\xi = \nu_\epsilon = \nu_\zeta = 0.5$, $\kappa = 2$, and $\nu_{z_1} = \nu_{z_2} = 0.15$.

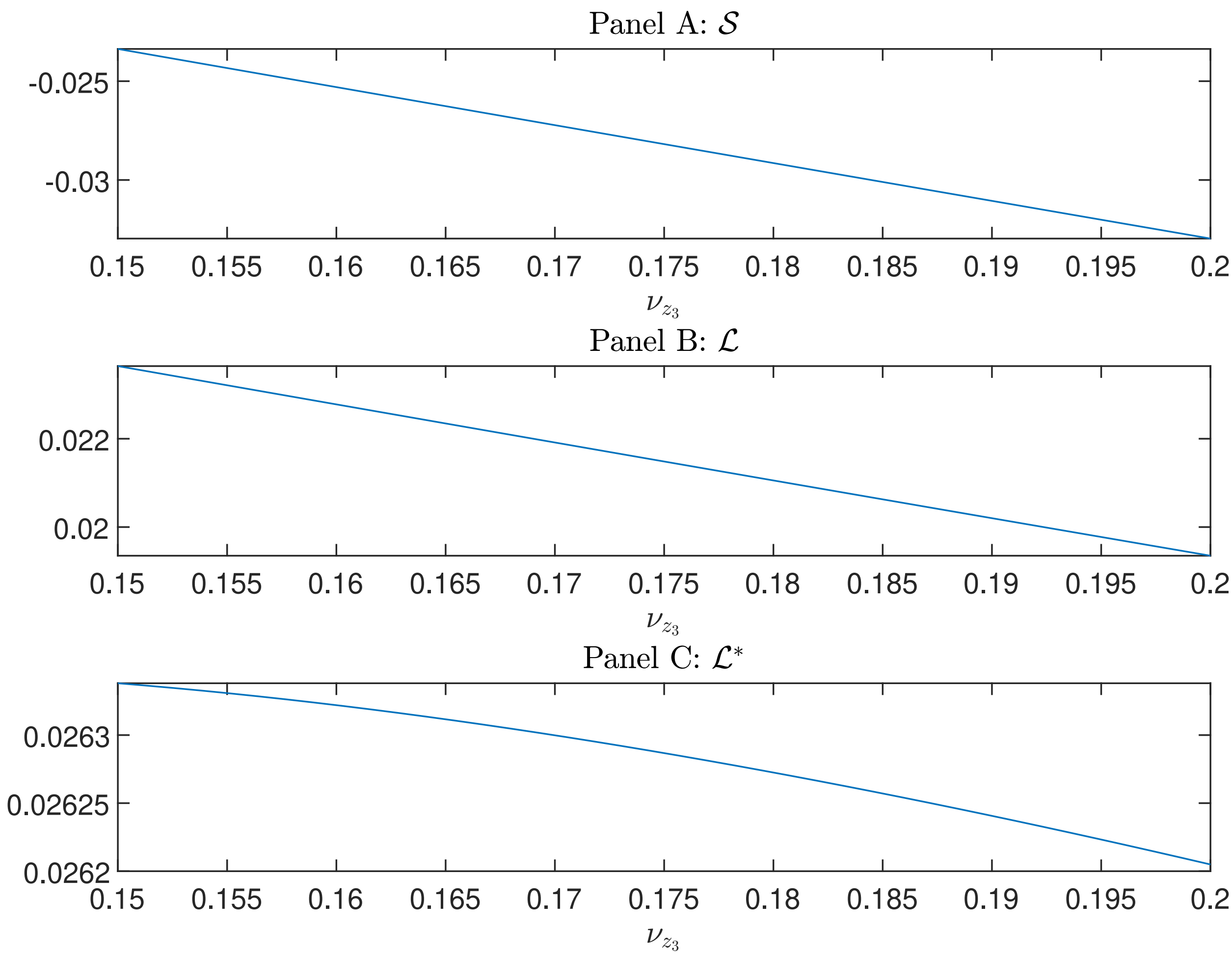


Table 1: The timeline of the model

Date 0 (prior date)	Date 1	Date 2	Date 3	Date 4 (terminal date)
Information:				
No information signals	A public signal $f = \theta + \xi + \epsilon + \zeta$ is released.	A public signal (earnings) $F = \theta + \xi + \epsilon$ is released.	Informed investors observe a private signal $s = \theta + \xi$; uninformed investors do not.	The stock pays a liquidation value $V = \theta$.
Beliefs:				
	Uninformed investors believe that f reveals only part of the payoff (i.e., $f = \theta_1 + \xi + \epsilon + \zeta$).	Uninformed investors believe that F reveals only part of the payoff (i.e., $F = \theta_1 + \xi + \epsilon$).	Uninformed investors believe that s reveals only part of the payoff (i.e., $s = \theta_1 + \xi$).	
	Informed investors assess the informativeness of f properly.	Informed investors assess the informativeness of F properly.	Informed investors assess the informativeness of s properly.	
Noise trades:				
	There is a noise demand z_1.	$(1-\mu)z_1$ of the Date-1 noise demand is unwound;	The remaining μz_1 of the Date-1 noise demand is unwound;	
		there is a new noise demand z_2.	$(1-\mu)z_2$ of the Date-2 noise demand is unwound;	
			there is a new noise demand z_3.	
Prices:				
P_0 is formed (and is non-stochastic).	P_1 is formed.	P_2 is formed.	P_3 is formed.	P_4 is formed (and equals θ).

Table 2: Monthly regression coefficients on lagged returns

This table presents the results of the following regression:

$$r_{i,t} = \rho_0 + \sum_{j=1}^{12} (\rho_j \times r_{i,t-j}) + \epsilon_{i,t},$$

where $r_{i,t}$ is return on stock i in month t. The table reports the regression coefficients and the corresponding t-statistics (in parentheses). The coefficients and the standard errors used to compute the t-statistics are estimated using the Fama-MacBeth method. The $\chi^2(12)$-statistic in the last column is computed under the null hypothesis that the slope coefficients ρ_1 through ρ_{12} are jointly equal to zero. The 10%, 5%, and 1% critical values for a $\chi^2(12)$-statistic are 18.5, 21.0, and 26.2, respectively. Panel A presents the results for U.S. stocks. The U.S. sample is comprised of common stocks on CRSP with share codes 10 or 11, excluding stocks with market capitalization below the 10% NYSE breakpoint and also stocks priced less than \$1 at the end of month $t-1$. The sample period is January 1931 to December 2020. Panel B presents the results for international stocks from the countries listed in Footnote 24. We obtain data on international stock returns (computed in USD) from Datastream. The sample excludes all stocks priced less than \$1 (USD) at the end of month $t-1$. The sample period is from January 1991 to December 2020.

Period		ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	ρ_6	ρ_7	ρ_8	ρ_9	ρ_{10}	ρ_{11}	ρ_{12}	$\sum_{i=2}^{12} \rho_i$	$\chi^2(12)$
Panel A: U.S. stocks															
193101 to 202012	Estimate	−0.0450	0.0022	0.0181	0.0094	0.0099	0.0142	0.0087	0.0015	0.0119	0.0082	0.0135	0.0181	0.1157	354.6
	(t-stat.)	(−13.54)	(0.76)	(6.28)	(3.43)	(3.42)	(4.66)	(3.14)	(0.58)	(5.06)	(3.57)	(5.68)	(7.64)	(7.01)	
193101 to 196012	Estimate	−0.0748	0.0012	0.0264	0.0096	0.0074	0.0189	0.0102	−0.0042	0.0065	0.0098	0.0173	0.0265	0.1297	181.1
	(t-stat.)	(−10.66)	(0.19)	(4.08)	(1.56)	(1.14)	(2.57)	(1.63)	(−0.72)	(1.23)	(1.96)	(3.20)	(4.94)	(3.43)	
196101 to 199012	Estimate	−0.0480	0.0027	0.0197	0.0175	0.0131	0.0175	0.0118	0.0079	0.0191	0.0112	0.0156	0.0267	0.1630	272.5
	(t-stat.)	(−9.72)	(0.63)	(4.78)	(4.30)	(3.07)	(4.60)	(3.03)	(2.10)	(5.37)	(3.25)	(4.62)	(7.96)	(7.01)	
199101 to 202012	Estimate	−0.0121	0.0027	0.0082	0.0012	0.0092	0.0061	0.0042	0.0008	0.0100	0.0035	0.0075	0.0011	0.0544	32.52
	(t-stat.)	(−2.69)	(0.61)	(2.07)	(0.33)	(2.42)	(1.60)	(1.08)	(0.22)	(3.37)	(1.09)	(2.39)	(0.35)	(2.51)	
Panel B: Non-U.S. stocks															
199101 to 202012	Estimate	−0.0158	0.0016	0.0102	0.0045	0.0050	0.0090	0.0052	0.0067	0.0124	0.0048	0.0075	0.0091	0.0761	96.28
	(t-stat.)	(−3.62)	(0.45)	(3.16)	(1.50)	(1.62)	(2.83)	(1.73)	(2.16)	(4.56)	(1.82)	(2.89)	(3.42)	(4.86)	

Table 3: Lagged return interacted with earnings announcement dummies

This table presents the estimates for the following regression:

$$r_{i,t} = \rho_0 + \sum_{j=1}^{12} (\rho_j \times r_{i,t-j}) + b \times EAD\ Dummy_{i,t-1} + \phi \times EAD\ Dummy_{i,t-1} \times r_{i,t-1} + \epsilon_{i,t},$$

where $r_{i,t}$ is the return on stock i in month t and $EAD\ Dummy_{i,t-1}$ is an indicator variable that equals 1 if stock i announces earnings in month $t-1$. The table reports the regression coefficients and the corresponding t-statistics (in parentheses). The coefficients and the standard errors used to compute the t-statistics are estimated using the Fama-MacBeth method. The U.S. sample is comprised of common stocks on CRSP with share codes 10 or 11, excluding stocks with market capitalization below the 10% NYSE breakpoint stocks that are in the smallest NYSE decile and also stocks priced less than $1 at the end of month $t-1$. We obtain earnings announcement dates from Compustat. The sample excludes stocks that do not have at least one earnings announcement date on Compustat over the previous 12 months. The sample period is from January 1972 to December 2020. The international sample is comprised of stocks from the countries listed in Footnote 24. We obtain data on international stock returns (computed in USD) from Datastream and earnings announcement dates from Worldscope. The sample excludes all stocks that do not have at least one earnings announcement date on Worldscope over the previous 12 months. The sample period is from June 1992 to December 2020.

	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	ρ_6	ρ_7	ρ_8	ρ_9	ρ_{10}	ρ_{11}	ρ_{12}	b	ϕ
Panel A: U.S. stocks (197201 to 202012)														
Estimate	−0.0367	0.0036	0.0126	0.006	0.0081	0.0095	0.0055	0.006	0.0148	0.0083	0.0103	0.0113	−0.0004	0.0246
(t-stat.)	(−8.59)	(1.11)	(4.06)	(2.10)	(2.75)	(3.21)	(2.03)	(2.17)	(5.84)	(3.24)	(4.13)	(4.37)	(−0.85)	(7.12)
Panel B: Non-U.S. stocks (199206 to 202012)														
Estimate	−0.0304	0.0027	0.0127	0.0069	0.0062	0.0115	0.0045	0.0088	0.0087	0.0034	0.0074	0.0093	0.0014	0.0208
(t-stat.)	(−6.14)	(0.59)	(3.53)	(1.74)	(1.84)	(2.96)	(1.18)	(2.53)	(2.53)	(1.06)	(2.14)	(2.54)	(1.19)	(3.99)

Table 4: Reversal and momentum profits across countries

This table presents the results of the following pooled cross-sectional regression:

$$REV_{j,t} = \alpha_0 + \alpha_1 MOM_{j,t} + u_{j,t},$$

where $REV_{j,t}$ and $MOM_{j,t}$ are short-term reversal and momentum profits in country j during month t. For calculating REV and MOM we form decile-based hedge portfolios based on past one-month and past two-to-12 month returns, and measure the average returns on these portfolios the subsequent month. The table reports the regression coefficients and the corresponding t-statistics (in parentheses). The sample period is January 1991 to December 2020. The sample excludes all stocks priced less than \$1 (USD) at the end of month $t-1$. We require countries to have 100 firms as of previous December (otherwise, portfolio returns are unreliable).

	(1)	(2)
	REV	REV
MOM	-0.166	-0.194
(t-stat.)	(-6.10)	(-6.35)
Constant	0.00715	0.00847
(t-stat.)	(19.72)	(6.24)
Month FE	Yes	No
No. of Obs.	10,325	11,045
Adj-R^2	0.170	0.049

Table 5: Time-series correlations between reversal and momentum profits
This table presents the time-series correlation between monthly momentum (MOM) and short-term reversal (REV) profits within each country. For calculating REV and MOM we form decile-based hedge portfolios based on past one-month and past two-to-12 month returns, and measure the average returns on these portfolios the subsequent month. The sample period is January 1991 to December 2020. The sample excludes all stocks priced less than $1 (USD) at the end of month $t-1$. We require countries to have 100 firms as of previous December (otherwise, portfolio returns are unreliable).

Name of country	Corr(MOM,REV)
Austria	−0.1269
Australia	−0.2388
Belgium	−0.3278
Brazil	0.0303
Canada	−0.2709
Switzerland	−0.4019
Chile	0.1112
China	−0.3402
Czech Republic	−0.0095
Germany	−0.4117
Denmark	−0.2765
Egypt	−0.3995
Spain	−0.2534
Finland	−0.2510
France	−0.3735
United Kingdom	−0.4199
Greece	−0.1618
Hong Kong	−0.1243
Israel	−0.2514
India	−0.1759
Italy	−0.4167
Japan	−0.3728
Korea	−0.2006
Malaysia	−0.3616
Netherlands	−0.3112
Norway	−0.3273
New Zealand	0.0504
Pakistan	−0.1040
Poland	−0.2684
Portugal	−0.1728
Russia	0.1792
Saudi Arabia	−0.0378
Sweden	−0.2497
Singapore	−0.4072
Thailand	−0.1384
Turkey	−0.1068
Taiwan	−0.1242
South Africa	−0.1064
United States	−0.2662
Average	−0.2158
Median	−0.2510
% negative	89.7%

Table 6: Reversal, momentum, and culture

Panel A presents summary statistics and correlations for the cultural attributes of individualism (IDV) and uncertainty avoidance (UAI) described in Hofstede (2001). Details of the data collection procedure appear in Chui, Titman, and Wei (2010). Panel B presents average momentum (MOM) and short-term reversal (REV) profits (in percentages per month) across countries split by the median values of UAI and IDV (with medians assigned to the below-median group). For calculating the reversal profit (REV) we form decile-based hedge portfolios based on past one-month return, and measure the return on these portfolios the subsequent month. For the momentum profit (MOM) we replace the past month's return with the past two-to-twelve months' return. Columns (1) - (3) and (5) to (7) in Panel C of the table present the results of the following pooled cross-sectional regression:

$$PRED_{j,t} = \alpha_0 + \alpha_1 CULTURE_j + u_{j,t},$$

where $PRED_{j,t}$ is the short-term reversal (momentum) profit in country j during month t. $CULTURE_j$ is one of two cultural attributes based on Hofstede (2011). IDV is individualism and UAI is uncertainty avoidance. The table reports the regression coefficients and the corresponding t-statistics (in parentheses). All slopes are multiplied by 10^3. The sample period is January 1991 to December 2020. The sample excludes all stocks priced less than $1 (USD) at the end of month $t-1$. We require countries to have 100 firms as of the previous December.

Panel A: Summary statistics

	IDV	UAI
Mean	51.24	63.73
Median	54.00	68.00
Std. Dev.	23.71	24.10
Corr(IDV, UAI)	−0.167	

Panel B: MOM and REV split by UAI and IDV medians

		MOM	REV
UAI	Above median	1.118	0.609
	Below median	1.615	0.233
IDV	Above median	1.650	0.506
	Below median	0.961	0.245

[Table continues on next page]

Table 6, continued

Panel C: Regressions						
	(1)	(2)	(3)	(4)	(5)	(6)
	REV	REV	REV	MOM	MOM	MOM
IDV	−0.0125		0.0272	0.183		0.168
(t-stat.)	(−0.36)		(0.77)	(4.93)		(4.43)
UAI		0.127	0.131		−0.0745	−0.0486
(t-stat.)		(5.19)	(5.24)		(−2.81)	(−1.79)
Constant	0.00477	−0.00394	−0.00575	0.00344	0.0185	0.00734
(t-stat.)	(2.34)	(−2.39)	(−2.01)	(1.56)	(10.42)	(2.37)
Month x Developed FE	Yes	Yes	Yes	Yes	Yes	Yes
No. of Obs.	9,785	9,785	9,785	9,785	9,785	9,785
Adj-R^2	0.163	0.166	0.166	0.241	0.239	0.241

Table 7: Return reversal and retail order flow

Columns (1) and (2) of this table present the results from subsets of the following regression:

$$r_{i,t} \quad = \quad \rho_0 + \rho_1 \times r_{i,t-1} + \rho_2 \times |Retail\ OIB|_{i,t-1} + \rho_3 \times r_{i,t-1} \times |Retail\ OIB|_{i,t-1} + \epsilon_{i,t},$$

where $r_{i,t}$ is the return on stock i in month t. Column (1) only uses the lagged return $r_{i,t-1}$ as the right-hand variable. For column (2), we compute $|Retail\ OIB|_{i,t}$ as follows: We first calculate

$$Net\ Retail\ Buy_{i,t} = \frac{Retail\ Buy_{i,t} - Retail\ Sell_{i,t}}{Retail\ Buy_{i,t} + Retail\ Sell_{i,t}},$$

where $Retail\ Buy_{i,t}$ and $Retail\ Sell_{i,t}$ are the number of shares of stock i bought and sold by small investors in month t, as reported in TAQ. We subtract the cross-sectional mean of $Net\ Retail\ Buy_{i,t}$ and then divide by the cross-sectional standard deviation to compute $Retail\ OIB_{i,t-1}$. The absolute value of this quantity is denoted by $|Retail\ OIB|_{i,t-1}$. Column (3) of the regression replaces $|Retail\ OIB|_{i,t}$ with the variable $\sigma(Retail\ OIB)_{i,t}$, where the latter is calculated as follows. We first compute

$$Net\ Retail\ Buy_{i,j,t} = \frac{Retail\ Buy_{i,j,t} - Retail\ Sell_{i,j,t}}{Retail\ Buy_{i,j,t} + Retail\ Sell_{i,j,t}},$$

where $Retail\ Buy_{i,j,t}$ and $Retail\ Sell_{i,j,t}$ are the number of shares of stock i bought and sold by retail investors on day j of month t, as reported in TAQ. The monthly standard deviation of this quantity is denoted by $\sigma(Retail\ OIB)_{i,t}$, which is then cross-sectionally standardized. The table reports the regression coefficients and the corresponding t-statistics (in parentheses). The coefficients and the standard errors used to compute the t-statistics are estimated using the Fama-MacBeth method. The sample is comprised of all common stocks on CRSP that we are able to match with TAQ. The sample period is from November 2006 to December 2021.

	(1)	(2)	(3)
$r_{i,t-1}$	−0.0212	−0.0102	−0.0181
(t-stat.)	(−2.83)	(−1.33)	(−2.61)
$\|Retail\ OIB\|_{i,t-1}$		0.0003	
(t-stat.)		(0.34)	
$r_{i,t-1} \times \|Retail\ OIB\|_{i,t-1}$		−0.0244	
(t-stat.)		(−3.18)	
$\sigma(Retail\ OIB)_{i,t-1}$			0.0007
(t-stat.)			(0.85)
$r_{i,t-1} \times \sigma(Retail\ OIB)_{i,t-1}$			−0.013
(t-stat.)			(−3.60)

Appendix A: Proofs

Proof of Proposition 1: Conjecture that the prices take the forms in Proposition 1. In what follows, we verify the conjectured prices and solve for the pricing parameters. We use backward induction. Informed investors have the information sets $\{f, z_1\}$ at Date 1, $\{f, F, z_1, z_2\}$ at Date 2, and $\{f, F, s, z_1, z_2, z_3\}$ at Date 3; they have no information at Date 0. Since uninformed investors learn $\omega = s + \delta z_3$ from the Date-3 price, they have the information sets $\{f, z_1\}$ at Date 1, $\{f, F, z_1, z_2\}$ at Date 2, and $\{f, F, \omega, z_1, z_2\}$ at Date 3; they have no information at Date 0.

Date 3: Note that s dominates f and F in providing information for the payoff θ. The i'th informed investor believes that $\theta|s \sim N(\gamma_\eta s, \nu_{\theta|s})$, where γ_η and $\nu_{\theta|s}$ are constants depending on the variance-covariance matrix of θ and s conditional on the investor's belief. Write the investor's wealth at Date 4 as $W_{i4} = W_{i3} + X_{i3}(\theta - P_3)$. The demand X_{i3} maximizes

$$\begin{aligned} E_{\eta 3}\left[U(W_{i4})\right] &= E_{\eta 3}\left[-\exp(-AW_{i4})\right] \\ &= E_{\eta 3}\left[-\exp\left[-AW_{i3} - AX_{i3}(\theta - P_3)\right]\right] \\ &= -\exp\left[-AW_{i3} - AX_{i3}\left(\gamma_\eta s - P_3\right) + 0.5A^2 X_{i3}^2 \nu_{\theta|s}\right], \end{aligned} \tag{A.1}$$

where $E_{\eta t}(.)$ indicates an expectation based on informed investors' beliefs and information set at Date t. The first order condition (f.o.c.) with respect to (w.r.t.) X_{i3} implies that the demand can be expressed as:

$$X_{\eta 3} = \frac{\gamma_\eta s - P_3}{A\nu_{\theta|s}}. \tag{A.2}$$

The second order condition obviously holds in the above case, and all other cases below, so that we omit referencing it in the rest of the proofs.

Since F dominates f in providing information for θ, the i'th uninformed investor believes that $\theta|(\omega, F) \sim N(\gamma_{\ell\omega}\omega + \gamma_{\ell F} F, \kappa_{\theta|\omega,F})$, where the γ's and $\kappa_{\theta|\omega,F}$ are constants depending on the variance-covariance matrix of θ, ω, and F based on the investor's belief.

The demand X_{i3} maximizes

$$\begin{aligned} E_{\ell 3}\left[U(W_{i4})\right] &= E_{\ell 3}\left[-\exp(-AW_{i4})\right] \\ &= E_{\ell 3}\left[-\exp\left[-AW_{i3} - AX_{i3}(\theta - P_3)\right]\right] \\ &= -\exp\left[-AW_{i3} - AX_{i3}\left(\gamma_{\ell\omega}\omega + \gamma_{\ell F}F - P_3\right) + 0.5A^2X_{i3}^2\kappa_{\theta|\omega,F}\right], \quad \text{(A.3)} \end{aligned}$$

where $E_{\ell t}(.)$ indicates taking an expectation based on uninformed investors' beliefs and information set at Date t. The f.o.c. w.r.t. X_{i3} implies that the demand can be expressed as:

$$X_{\ell 3} = \frac{\gamma_{\ell\omega}\omega + \gamma_{\ell F}F - P_3}{A\kappa_{\theta|\omega,F}}. \quad \text{(A.4)}$$

The market clearing condition, $\lambda X_{\eta 3} + (1-\lambda)X_{\ell 3} + z_3 + \mu z_2 = 0$, implies that the Date-3 price takes the form:

$$P_3 = \gamma_1\omega + \gamma_2 F + \gamma_3\mu z_2, \quad \text{(A.5)}$$

where the parameters

$$\gamma_1 = \frac{\lambda\nu_{\theta|s}^{-1}\gamma_\eta + (1-\lambda)\kappa_{\theta|\omega,F}^{-1}\gamma_{\ell\omega}}{\lambda\nu_{\theta|s}^{-1} + (1-\lambda)\kappa_{\theta|\omega,F}^{-1}}, \quad \gamma_2 = \frac{(1-\lambda)\kappa_{\theta|\omega,F}^{-1}\gamma_{\ell F}}{\lambda\nu_{\theta|s}^{-1} + (1-\lambda)\kappa_{\theta|\omega,F}^{-1}},$$
$$\gamma_3 = \frac{A}{\lambda\nu_{\theta|s}^{-1} + (1-\lambda)\kappa_{\theta|\omega,F}^{-1}}, \quad \text{and} \quad \delta = \frac{A}{\lambda\nu_{\theta|s}^{-1}\gamma_\eta}.$$

Denote $\mathcal{S} = \begin{pmatrix} s \\ z_3 \end{pmatrix}$ and $\Psi = \begin{pmatrix} F \\ z_2 \end{pmatrix}$. From Equations (A.1), (A.2), and (A.5), the i'th informed investor's value function at Date 3 can be expressed as

$$\begin{aligned} J_{\eta 3}(W_{i3}, \mathcal{S}, \Psi) &= E_{\eta 3}\left[U(W_{i4})\right] \\ &= -\exp\left[-AW_{i3} - 0.5\left(\gamma_\eta s - P_3\right)^2/\nu_{\theta|s}\right] \\ &= -\exp\left(-AW_{i3} - 0.5\mathcal{S}^T d_1\mathcal{S} - \mathcal{S}^T d_{12}\Psi - 0.5\Psi^T d_2\Psi\right), \quad \text{(A.6)} \end{aligned}$$

where the d's are constants. From Equations (A.3), (A.4), and (A.5), the i'th uninformed investor's Date 3 value function is

$$\begin{aligned} J_{\ell 3}(W_{i3}, \mathcal{S}, \Psi) &= E_{\ell 3}\left[U(W_{i4})\right] \\ &= -\exp\left[-AW_{i3} - 0.5(\gamma_{\ell\omega}\omega + \gamma_{\ell F}F - P_3)^2/\kappa_{\theta|\omega,F}\right] \\ &= -\exp\left(-AW_{i3} - 0.5\mathcal{S}^T e_1\mathcal{S} - \mathcal{S}^T e_{12}\Psi - 0.5\Psi^T e_2\Psi\right), \quad \text{(A.7)} \end{aligned}$$

where the e's are constants.

Date 2: Express P_3 in Equation (A.5) as

$$P_3 = C^T\mathcal{S} + \gamma_2 F + \gamma_3\mu z_2,$$

where C is a constant. The i'th informed investor believes that $\mathcal{S}|F \sim N(\beta_\eta F, \nu_{\mathcal{S}|F})$, where β_η and $\nu_{\mathcal{S}|F}$ are constants depending on the variance-covariance matrix of $\mathcal{S}$ and F based on the investor's belief. Consider the investor's value function in Equation (A.6). Write the wealth at Date 3 as $W_{i3} = W_{i2} + X_{i2}(P_3 - P_2)$. The demand X_{i2} maximizes

$$\begin{aligned}
& E_{\eta 2}\left[J_{\eta 3}(W_{i3}, \mathcal{S}, \Psi)\right] \qquad \text{(A.8)}\\
= \;& E_{\eta 2}\left[-\exp\left(-AW_{i3} - 0.5\mathcal{S}^T d_1\mathcal{S} - \mathcal{S}^T d_{12}\Psi - 0.5\Psi^T d_2\Psi\right)\right]\\
= \;& E_{\eta 2}\Big[-\exp\Big[-AW_{i2} - AX_{i2}\left(C^T\mathcal{S} + \gamma_2 F + \gamma_3\mu z_2 - P_2\right)\\
& -0.5\mathcal{S}^T d_1\mathcal{S} - \mathcal{S}^T d_{12}\Psi - 0.5\Psi^T d_2\Psi\Big]\Big]\\
\propto \;& -\exp\Big[-AW_{i2} - AX_{i2}\left(\gamma_2 F + \gamma_3\mu z_2 - P_2\right) + 0.5\left(AX_{i2}C + d_{12}\Psi - \nu_{\mathcal{S}|F}^{-1}\beta_\eta F\right)^T\\
& (\nu_{\mathcal{S}|F}^{-1} + d_1)^{-1}\left(AX_{i2}C + d_{12}\Psi - \nu_{\mathcal{S}|F}^{-1}\beta_\eta F\right) - 0.5(\beta_\eta F)^T\nu_{\mathcal{S}|F}^{-1}(\beta_\eta F) - 0.5\Psi^T d_2\Psi\Big].
\end{aligned}$$

Here and in what follows, we repeatedly use the fact in Footnote 40.[40] The f.o.c. w.r.t. X_{i2} implies that the demand can be expressed as

$$\begin{aligned}
X_{\eta 2} &= \frac{\gamma_2 F + \gamma_3\mu z_2 - P_2 - C^T(\nu_{\mathcal{S}|F}^{-1} + d_1)^{-1}(d_{12}\Psi - \nu_{\mathcal{S}|F}^{-1}\beta_\eta F)}{AC^T(\nu_{\mathcal{S}|F}^{-1} + d_1)^{-1}C}\\
&= \frac{\beta_{\eta 1}F + \gamma_3\mu z_2 - \beta_{\eta 3}z_2 - P_2}{A\Sigma_{\eta 2}}, \qquad \text{(A.9)}
\end{aligned}$$

where the β's and $\Sigma_{\eta 2}$ are constants.

The i'th uninformed investor believes that $\mathcal{S}|F \sim N(\beta_\ell F, \kappa_{\mathcal{S}|F})$, where β_ℓ and $\kappa_{\mathcal{S}|F}$ are constants depending on the variance-covariance matrix of $\mathcal{S}$ and F based on the investor's belief. Consider the investor's value function in Equation (A.7). The demand X_{i2}

[40] x can be a vector or scalar, where n is the dimension of x. If $x \sim N(\bar{x}, \Sigma)$, then

$$E\left[\exp(\varrho^T x - 0.5x^T\Lambda x)\right] = \frac{\exp\left[0.5(\varrho + \Sigma^{-1}\bar{x})^T(\Sigma^{-1} + \Lambda)^{-1}(\varrho + \Sigma^{-1}\bar{x}) - 0.5\bar{x}^T\Sigma^{-1}\bar{x}\right]}{(|\Sigma^{-1} + \Lambda||\Sigma|)^{0.5}}.$$

maximizes

$$\begin{aligned}
& E_{\ell 2}\left[J_{\ell 3}(W_{i3}, \mathcal{S}, \Psi)\right] \qquad \text{(A.10)} \\
= \; & E_{\ell 2}\left[-\exp\left(-AW_{i3} - 0.5\mathcal{S}^T e_1 \mathcal{S} - \mathcal{S}^T e_{12}\Psi - 0.5\Psi^T e_2 \Psi\right)\right] \\
= \; & E_{\ell 2}\Big[-\exp\Big[-AW_{i2} - AX_{i2}\left(C^T\mathcal{S} + \gamma_2 F + \gamma_3 \mu z_2 - P_2\right) \\
& -0.5\mathcal{S}^T e_1 \mathcal{S} - \mathcal{S}^T e_{12}\Psi - 0.5\Psi^T e_2 \Psi\Big]\Big] \\
\propto \; & -\exp\Big[-AW_{i2} - AX_{i2}\left(\gamma_2 F + \gamma_3 \mu z_2 - P_2\right) + 0.5\left(AX_{i2}C + e_{12}\Psi - \kappa_{\mathcal{S}|F}^{-1}\beta_\ell F\right)^T \\
& (\kappa_{\mathcal{S}|F}^{-1} + e_1)^{-1}\left(AX_{i2}C + e_{12}\Psi - \kappa_{\mathcal{S}|F}^{-1}\beta_\ell F\right) - 0.5(\beta_\ell F)^T \kappa_{\mathcal{S}|F}^{-1}(\beta_\ell F) - 0.5\Psi^T e_2 \Psi\Big].
\end{aligned}$$

The f.o.c. w.r.t. X_{i2} implies that the demand can be expressed as

$$\begin{aligned}
X_{\ell 2} &= \frac{\gamma_2 F + \gamma_3 \mu z_2 - P_2 - C^T(\kappa_{\mathcal{S}|F}^{-1} + e_1)^{-1}(e_{12}\Psi - \kappa_{\mathcal{S}|F}^{-1}\beta_\ell F)}{AC^T(\kappa_{\mathcal{S}|F}^{-1} + e_1)^{-1}C} \\
&= \frac{\beta_{\ell 1}F + \gamma_3 \mu z_2 - \beta_{\ell 3} z_2 - P_2}{A\Sigma_{\ell 2}}, \qquad \text{(A.11)}
\end{aligned}$$

where the β's and $\Sigma_{\ell 2}$ are constants.

The market clearing condition, $\lambda X_{\eta 2} + (1-\lambda)X_{\ell 2} + z_2 + \mu z_1 = 0$, implies that the Date-2 price P_2 takes the form

$$P_2 = \beta_1 F + \gamma_3 \mu z_2 + \beta_2(z_2 + \mu z_1) - \beta_3 z_2, \qquad \text{(A.12)}$$

where the parameters

$$\beta_1 = \frac{\lambda \Sigma_{\eta 2}{}^{-1}\beta_{\eta 1} + (1-\lambda)\Sigma_{\ell 2}{}^{-1}\beta_{\ell 1}}{\lambda \Sigma_{\eta 2}{}^{-1} + (1-\lambda)\Sigma_{\ell 2}{}^{-1}}, \quad \beta_2 = \frac{A}{\lambda \Sigma_{\eta 2}{}^{-1} + (1-\lambda)\Sigma_{\ell 2}{}^{-1}}, \quad \text{and}$$
$$\beta_3 = \frac{\lambda \Sigma_{\eta 2}{}^{-1}\beta_{\eta 3} + (1-\lambda)\Sigma_{\ell 2}{}^{-1}\beta_{\ell 3}}{\lambda \Sigma_{\eta 2}{}^{-1} + (1-\lambda)\Sigma_{\ell 2}{}^{-1}}.$$

From Equations (A.8), (A.9), and (A.12), the i'th informed investor's value function at Date 2 can be expressed as

$$\begin{aligned}
& J_{\eta 2}(W_{i2}, \Psi, z_1) = E_{\eta 2}\left[J_{\eta 3}(W_{i3}, \mathcal{S}, \Psi)\right] \\
\propto \; & -\exp\Big[-AW_{i2} - 0.5\frac{(\beta_{\eta 1}F + \gamma_3 \mu z_2 - \beta_{\eta 3} z_2 - P_2)^2}{\Sigma_{\eta 2}} + 0.5\left(d_{12}\Psi - \nu_{\mathcal{S}|F}^{-1}\beta_\eta F\right)^T \\
& (\nu_{\mathcal{S}|F}^{-1} + d_1)^{-1}\left(d_{12}\Psi - \nu_{\mathcal{S}|F}^{-1}\beta_\eta F\right) - 0.5(\beta_\eta F)^T \nu_{\mathcal{S}|F}^{-1}(\beta_\eta F) - 0.5\Psi^T d_2 \Psi\Big] \\
= \; & -\exp\left(-AW_{i2} - 0.5\Psi^T h_1 \Psi - \Psi^T h_{12} z_1 - 0.5 h_2 z_1^2\right), \qquad \text{(A.13)}
\end{aligned}$$

where the h's are constants. From Equations (A.10), (A.11), and (A.12), the i'th uninformed investor's value function at Date 2 can be expressed as

$$
\begin{aligned}
& J_{\ell 2}(W_{i2}, \Psi, z_1) = E_{\ell 2}\left[J_{\ell 3}(W_{i3}, \mathcal{S}, \Psi)\right] \\
\propto \; & -\exp\Big[-AW_{i2} - 0.5\frac{(\beta_{\ell 1}F + \gamma_3 \mu z_2 - \beta_{\ell 3} z_2 - P_2)^2}{\Sigma_{\ell 2}} + 0.5\left(e_{12}\,\Psi - \kappa_{\mathcal{S}|F}^{-1}\beta_\ell F\right)^T \\
& (\kappa_{\mathcal{S}|F}^{-1} + e_1)^{-1}\left(e_{12}\,\Psi - \kappa_{\mathcal{S}|F}^{-1}\beta_\ell F\right) - 0.5(\beta_\ell F)^T \kappa_{\mathcal{S}|F}^{-1}(\beta_\ell F) - 0.5\,\Psi^T e_2\,\Psi\Big] \\
= \; & -\exp\left(-AW_{i2} - 0.5\,\Psi^T g_1\,\Psi - \Psi^T g_{12} z_1 - 0.5 g_2 z_1^2\right), \qquad \text{(A.14)}
\end{aligned}
$$

where the g's are constants.

Date 1: Express P_2 in Equation (A.12) as

$$
P_2 = B^T \Psi + \beta_2 \mu z_1,
$$

where B is a constant. The i'th informed investor believes that $\Psi|f \sim N(\alpha_\eta f, \nu_{\Psi|f})$, where α_η and $\nu_{\Psi|f}$ are constants depending on the variance-covariance matrix of Ψ and f based on the investor's belief. Consider the investor's value function in Equation (A.13). Write the wealth at Date 2 as $W_{i2} = W_{i1} + X_{i1}(P_2 - P_1)$. The demand X_{i1} maximizes

$$
\begin{aligned}
& E_{\eta 1}\left[J_{\eta 2}(W_{i2}, \Psi, z_1)\right] \\
\propto \; & E_{\eta 1}\left[-\exp\left(-AW_{i2} - 0.5\,\Psi^T h_1\,\Psi - \Psi^T h_{12} z_1 - 0.5 h_2 z_1^2\right)\right] \\
= \; & E_{\eta 1}\Big[-\exp\Big[-AW_{i1} - AX_{i1}\left(B^T \Psi + \beta_2 \mu z_1 - P_1\right) \\
& -0.5\,\Psi^T h_1\,\Psi - \Psi^T h_{12} z_1 - 0.5 h_2 z_1^2\Big]\Big] \\
\propto \; & -\exp\Big[-AW_{i1} - AX_{i1}\left(\beta_2 \mu z_1 - P_1\right) + 0.5\left(AX_{i1}B + h_{12} z_1 - \nu_{\Psi|f}^{-1}\alpha_\eta f\right)^T \\
& (\nu_{\Psi|f}^{-1} + h_1)^{-1}\left(AX_{i1}B + h_{12} z_1 - \nu_{\Psi|f}^{-1}\alpha_\eta f\right) - 0.5(\alpha_\eta f)^T \nu_{\Psi|f}^{-1}(\alpha_\eta f) - 0.5 h_2 z_1^2\Big] \qquad \text{(A.15)}
\end{aligned}
$$

The f.o.c. w.r.t. X_{i1} implies that the demand can be expressed as

$$
\begin{aligned}
X_{\eta 1} &= \frac{\beta_2 \mu z_1 - P_1 - B^T(\nu_{\Psi|f}^{-1} + h_1)^{-1}(h_{12} z_1 - \nu_{\Psi|f}^{-1}\alpha_\eta f)}{AB^T(\nu_{\Psi|f}^{-1} + h_1)^{-1}B} \\
&= \frac{\alpha_{\eta 1} f + \beta_2 \mu z_1 - \alpha_{\eta 3} z_1 - P_1}{A\Sigma_{\eta 1}}, \qquad \text{(A.16)}
\end{aligned}
$$

where the α's and $\Sigma_{\eta 1}$ are constants.

The i'th uninformed investor believes that $\Psi|f \sim N(\alpha_\ell f, \kappa_{\Psi|f})$, where α_ℓ and $\kappa_{\Psi|f}$ are constants depending on the variance-covariance matrix of Ψ and f based on the investor's belief. Consider the investor's value function in Equation (A.14). The demand X_{i1} maximizes

$$\begin{aligned}
& E_{\ell 1}\left[J_{\ell 2}(W_{i2}, \Psi, z_1)\right] \\
\propto\ & E_{\ell 1}\left[-\exp\left(-AW_{i2} - 0.5\,\Psi^T g_1\,\Psi - \Psi^T g_{12} z_1 - 0.5 g_2 z_1^2\right)\right] \\
=\ & E_{\ell 1}\Big[-\exp\Big[-AW_{i1} - AX_{i1}\left(B^T\Psi + \beta_2\mu z_1 - P_1\right) \\
& -0.5\,\Psi^T g_1\,\Psi - \Psi^T g_{12} z_1 - 0.5 g_2 z_1^2\Big]\Big] \\
\propto\ & -\exp\Big[-AW_{i1} - AX_{i1}\left(\beta_2\mu z_1 - P_1\right) + 0.5\left(AX_{i1}B + g_{12}z_1 - \kappa_{\Psi|f}^{-1}\alpha_\ell f\right)^T \\
& (\kappa_{\Psi|f}^{-1} + g_1)^{-1}\left(AX_{i1}B + g_{12}z_1 - \kappa_{\Psi|f}^{-1}\alpha_\ell f\right) - 0.5(\alpha_\ell f)^T\kappa_{\Psi|f}^{-1}(\alpha_\ell f) - 0.5 g_2 z_1^2\Big]
\end{aligned} \tag{A.17}$$

The f.o.c. w.r.t. X_{i1} implies that the demand can be expressed as

$$\begin{aligned}
X_{\ell 1} &= \frac{\beta_2\mu z_1 - P_1 - B^T(\kappa_{\Psi|f}^{-1} + g_1)^{-1}(g_{12}z_1 - \kappa_{\Psi|f}^{-1}\alpha_\ell f)}{AB^T(\kappa_{\Psi|f}^{-1} + g_1)^{-1}B} \\
&= \frac{\alpha_{\ell 1} f + \beta_2\mu z_1 - \alpha_{\ell 3} z_1 - P_1}{A\Sigma_{\ell 1}},
\end{aligned} \tag{A.18}$$

where the α's and $\Sigma_{\ell 1}$ are constants.

The market clearing condition, $\lambda X_{\eta 1} + (1-\lambda)X_{\ell 1} + z_1 = 0$, implies that the Date-1 price P_1 takes the form

$$P_1 = \alpha_1 f + \beta_2\mu z_1 + \alpha_2 z_1, \tag{A.19}$$

where the parameters

$$\begin{aligned}
\alpha_1 &= \frac{\lambda\Sigma_{\eta 1}{}^{-1}\alpha_{\eta 1} + (1-\lambda)\Sigma_{\ell 1}{}^{-1}\alpha_{\ell 1}}{\lambda\Sigma_{\eta 1}{}^{-1} + (1-\lambda)\Sigma_{\ell 1}{}^{-1}}, \\
\alpha_2 &= \frac{A}{\lambda\Sigma_{\eta 1}{}^{-1} + (1-\lambda)\Sigma_{\ell 1}{}^{-1}} - \frac{\lambda\Sigma_{\eta 1}{}^{-1}\alpha_{\eta 3} + (1-\lambda)\Sigma_{\ell 1}{}^{-1}\alpha_{\ell 3}}{\lambda\Sigma_{\eta 1}{}^{-1} + (1-\lambda)\Sigma_{\ell 1}{}^{-1}}.
\end{aligned}$$

Denote $\psi = \begin{pmatrix} f \\ z_1 \end{pmatrix}$. From Equations (A.15), (A.16), and (A.19), the i'th informed investor's value function at Date 1 can be expressed as

$$\begin{aligned} & J_{\eta 1}(W_{i1}, \psi) = E_{\eta 1}\left[J_{\eta 2}(W_{i2}, \psi, z_1)\right] \\ \propto \quad & -\exp\Big[-AW_{i1} - 0.5\frac{(\alpha_{\eta 1} f + \beta_2 \mu z_1 - \alpha_{\eta 3} z_1 - P_1)^2}{\Sigma_{\eta 1}} + 0.5\left(h_{12} z_1 - \nu_{\psi|f}\alpha_\eta f\right)^T \\ & (\nu_{\psi|f}^{-1} + h_1)^{-1}\left(h_{12} z_1 - \nu_{\psi|f}\alpha_\eta f\right) - 0.6(\alpha_\eta f)^T \nu_{\psi|f}(\alpha_\eta f) - 0.5 h_2 z_1^2\Big] \\ = \quad & -\exp\left(-AW_{i1} - 0.5\psi^T q_\eta \psi^2\right), \end{aligned} \tag{A.20}$$

where q_η is a constant. We can similarly show that the i'th uninformed investor's value function at Date 1 can be expressed as

$$J_\ell(W_{i1}, z_1) \quad \propto \quad -\exp\left(-AW_{i1} - 0.5\psi^T q_\ell \psi\right),$$

where q_ℓ is a constant.

Date 0: Express P_1 in Equation (A.19) as

$$P_1 = Q\psi,$$

where Q is a constant. Consider the i'th informed investor's value function in Equation (A.20). Write the wealth at Date 1 as $W_{i1} = W_{i0} + X_{i0}(P_1 - P_0)$. The demand X_{i0} maximizes

$$\begin{aligned} & E_{\eta 0}\left[J_{\eta 1}(W_{i1}, \psi)\right] \propto E_{\eta 0}\left[-\exp\left(-AW_{i1} - 0.5\psi^T q_\eta \psi\right)\right] \\ = \quad & E_{\eta 0}\Big[-\exp\Big[-AW_{i0} - AX_{i0}\left(Q\psi - P_0\right) - 0.5\psi^T q_\eta \psi\Big]\Big] \\ \propto \quad & -\exp\Big[-AW_{i0} - AX_{i0}\left(-P_1\right) + 0.5\left(AX_{i0}Q\right)^T\left(\nu_\psi^{-1} + q_\eta\right)^{-1}\left(AX_{i0}Q\right)^2\Big]. \end{aligned}$$

The f.o.c. w.r.t. X_{i1} implies that the demand $X_{\eta 0} \propto -P_0$. We can use a similar analysis to show that the i'th uninformed investor's demand $X_{\ell 0} \propto -P_0$. The market clearing condition, $\lambda X_{\eta 0} + (1-\lambda)X_{\ell 0} = 0$, implies that the Date-0 price $P_0 = 0$. □

Proof of Lemma 1: The proof of this Lemma uses notation set out just prior to the statement of Proposition 2. Specifically, let the subscript ℓ indicate that the expectation is based on uninformed beliefs, and denote $E_\ell(\theta|s) \equiv \nu_{\theta_1}\kappa_s^{-1}s$ and $\kappa_{\theta|s} \equiv \nu_\theta - \nu_{\theta_1}^2\kappa_s^{-1}$ as the mean and variance of θ conditional on s. Further, let $\kappa_{E_\ell(\theta|s)|F} \equiv (\nu_{\theta_1}^2/\kappa_s)(1-\kappa_s\kappa_F^{-1})$ be the variance of $E_\ell(\theta|s)$ conditional on F. We use backward induction to solve for the prices. Note that the net noise demands at Dates 1, 2, and 3 are, respectively, z_1, $z_2+\mu z_1$, and μz_2.

Date 3: Noting that uninformed investors also learn s, we can use the same derivation as in the proof of Proposition 1 to show that the i'th uninformed investor chooses the demand X_{i3} to maximize

$$E_{\ell 3}\left[U(W_{i4})\right] \;=\; -\exp\left[-AW_{i3} - AX_{i3}\left(c_\ell s - P_3\right) + 0.5A^2X_{i3}^2\kappa_{\theta|s}\right],$$

where $c_\ell = \nu_{\theta_1}/\kappa_s$. The f.o.c. w.r.t. X_{i3} implies that the demand can be expressed as

$$X_{\ell 3} = \frac{c_\ell s - P_3}{A\kappa_{\theta|s}}.$$

The market clearing condition, $X_{\ell 3} + \mu z_2 = 0$, implies that the Date-3 price takes the form

$$P_3 = c_\ell s + c_3\mu z_2,$$

where $c_3 = A\kappa_{\theta|s} = A(\nu_\theta - \nu_{\theta_1}^2\kappa_s^{-1})$. It follows that the i'th uninformed investor's value function at Date 3 can be expressed as

$$J_{\ell 3}(W_{i3}, z_2) = E_{\ell 3}\left[U(W_{i4})\right] = -\exp\left[-AW_{i3} - 0.5Ac_3(\mu z_2)^2\right].$$

Date 2: Write the i'th uninformed investor's wealth at Date 3 as $W_{i3} = W_{i2} + X_{i2}(P_3 - P_2)$. The demand X_{i2} maximizes

$$\begin{aligned}
& E_{\ell 2}\left[J_{\ell 3}(W_{i3}, z_2)\right] \\
= \;& E_{\ell 2}\left[-\exp\left[-AW_{i3} - 0.5Ac_3(\mu z_2)^2\right]\right] \\
= \;& E_{\ell 2}\Big[-\exp\Big[-AW_{i2} - AX_{i2}\left(c_\ell s + c_3\mu z_2 - P_2\right) - 0.5Ac_3(\mu z_2)^2\Big]\Big] \\
\propto \;& -\exp\Big[-AW_{i2} - AX_{i2}\left(c_\ell E_\ell(s|F) + c_3\mu z_2 - P_2\right) + 0.5(AX_{i2}c_\ell)^2\kappa_{s|F} - 0.5Ac_3(\mu z_2)^2\Big].
\end{aligned}$$

The f.o.c. w.r.t. X_{i2} implies that the demand can be expressed as

$$X_{\ell 2} \;=\; \frac{c_\ell E_\ell(s|F) + c_3\mu z_2 - P_2}{Ac_\ell^2\kappa_{s|F}} = \frac{\nu_{\theta_1}\kappa_F^{-1}F + c_3\mu z_2 - P_2}{Ac_\ell^2\kappa_{s|F}}.$$

The market clearing condition, $X_{\ell 2} + z_2 + \mu z_1 = 0$, implies that the Date-2 price P_2 takes the form

$$P_2 = \frac{\nu_{\theta_1}}{\kappa_F} F + c_3 \mu z_2 + b_2(z_2 + \mu z_1),$$

where $b_2 = A c_\ell^2 \kappa_{s|F} = A(\nu_{\theta_1}^2/\kappa_s)(1 - \kappa_s \kappa_F^{-1})$. The i'th uninformed investor's value function at Date 2 can be expressed as

$$\begin{aligned} & J_{\ell 2}(W_{i2}, z_2, z_1) = E_{\ell 2}\left[J_{\ell 3}(W_{i3}, z_2)\right] \\ \propto \; & -\exp\left[-AW_{i2} - 0.5 A b_2 (z_2 + \mu z_1)^2 - 0.5 A c_3 (\mu z_2)^2\right]. \end{aligned}$$

Date 1: Write the i'th uninformed investor's wealth at Date 2 as $W_{i2} = W_{i1} + X_{i1}(P_2 - P_1)$. The demand X_{i1} maximizes

$$\begin{aligned} & E_{\ell 1}\left[J_{\ell 2}(W_{i2}, z_2, z_1)\right] \\ \propto \; & E_{\ell 1}\left[-\exp\left(-AW_{i2} - 0.5 A b_2 (z_2 + \mu z_1)^2 - 0.5 A c_3 (\mu z_2)^2\right)\right] \\ = \; & E_{\ell 1}\Bigg[-\exp\Bigg[-AW_{i1} - AX_{i1}\left(\frac{\nu_{\theta_1}}{\kappa_F} F + c_3 \mu z_2 + b_2(z_2 + \mu z_1) - P_1\right) \\ & -0.5 A b_2 (z_2 + \mu z_1)^2 - 0.5 A c_3 (\mu z_2)^2\Bigg]\Bigg] \\ \propto \; & -\exp\Bigg[-AW_{i1} - AX_{i1}\left(\frac{\nu_{\theta_1}}{\kappa_F} E_\ell(F|f) + b_2 \mu z_1 - P_1\right) + 0.5 A^2 X_{i1}^2 \left(\frac{\nu_{\theta_1}}{\kappa_F}\right)^2 \kappa_{F|f} \\ & + 0.5 \frac{\left[AX_{i1}(b_2 + c_3 \mu) + A b_2 \mu z_1\right]^2}{\nu_z^{-1} + A b_2 + A c_3 \mu^2} - 0.5 A b_2 (\mu z_1)^2\Bigg]. \end{aligned}$$

The f.o.c. w.r.t. X_{i1} implies that the demand can be expressed as

$$X_{\ell 1} = \frac{\nu_{\theta_1} {\kappa_F}^{-1} E_\ell(F|f) + b_2 \mu z_1 - P_1 - a_2 z_1}{a_1} = \frac{\nu_{\theta_1} {\kappa_f}^{-1} f + b_2 \mu z_1 - P_1 - a_2 z_1}{a_1},$$

where

$$a_1 = A \frac{\nu_{\theta_1}^2}{\kappa_F}\left(1 - \frac{\kappa_F}{\kappa_f}\right) + A \frac{(b_2 + c_3 \mu)^2}{\nu_z^{-1} + A b_2 + A c_3 \mu^2}, \quad \text{and} \quad a_2 = A \frac{(b_2 + c_3 \mu) b_2}{\nu_z^{-1} + A b_2 + A c_3 \mu^2} \mu.$$

The market clearing condition, $X_{\ell 1} + z_1 = 0$, implies that the Date-1 price P_1 takes the form

$$P_1 = \frac{\nu_{\theta_1}}{\kappa_f} f + b_2 \mu z_1 + (a_1 - a_2) z_1.$$

It is straightforward to show that if $0 \leq \mu \leq 1$, then $a \equiv a_1 - a_2 > 0$. We can use a similar derivation as in the proof of Proposition 1 to show that here $P_0 = 0$. $\square$

Proof of Proposition 2: Denote

$$\Delta \equiv A(\nu_{\theta_1}^2/\kappa_s)(1 - \kappa_s \kappa_F^{-1}).$$

Further, let $\mathcal{S}^K$ and $\mathcal{L}^K$ respectively denote the values of $\mathcal{S}$ and $\mathcal{L}$ when $\nu_z = 0$. We then have

$$\begin{aligned} \mathcal{S}^K &\equiv \text{Cov}\left[E_\ell(\theta|f), E_\ell(\theta|F) - E_\ell(\theta|f)\right] + \text{Cov}\left[E_\ell(\theta|F) - E_\ell(\theta|f), E_\ell(\theta|s) - E_\ell(\theta|F)\right] \\ &\quad + \text{Cov}\left[E_\ell(\theta|s) - E_\ell(\theta|F), \theta - E_\ell(\theta|s)\right]. \quad \text{(A.21)} \\ \mathcal{L}^K &\equiv \frac{\nu_{\theta_1}}{\kappa_F}\left(\nu_\theta - \frac{\nu_{\theta_1}}{\kappa_F}\nu_F\right). \quad \text{(A.22)} \end{aligned}$$

We assume a range for the scale of noise trades, ν_z, that represents a sufficient condition for short-term reversals (which obtain for high ν_z) as well as longer term momentum (which require low ν_z):[41]

$$\nu_z \in \left[\frac{\max\left(\mathcal{S}^K, \text{Cov}\left(E_\ell(\theta|F), E_\ell(\theta|s) - E_\ell(\theta|F)\right)\right)}{\Delta^2}, \frac{\mathcal{L}^K}{\Delta^2}\right]. \quad \text{(A.23)}$$

Condition (A.23) implies that z_1 and z_2 have a significant but modest scale.

For Part (i) of this proposition, note that the momentum parameter

$$\mathcal{L} = \text{Cov}(P_2 - P_0, P_4 - P_2) = \mathcal{L}^K - (c_3\mu + b_2)^2\nu_z - (b_2\mu)^2\nu_z, \quad \text{(A.24)}$$

where

$$\mathcal{L}^K = \text{Cov}(E_\ell(\theta|F), \theta - E_\ell(\theta|F)) = \text{Cov}\left(\frac{\nu_{\theta_1}}{\kappa_F}F, \theta - \frac{\nu_{\theta_1}}{\kappa_F}F\right)$$

It is obvious that $\mathcal{L} < \mathcal{L}^K$, that $\mathcal{L}$ decreases in μ, and that if $\mu = 0$, then $\mathcal{L} > 0$ under Condition (A.23). Since all endogenous parameters are continuous in μ, Part (i) follows.

[41] A sufficient condition for Condition (A.23) to specify a non-empty set of parameter values is:

$$0 < \frac{\nu_1}{\kappa_f}\left(\frac{\nu_\zeta}{\kappa_f} - \frac{\nu_\epsilon}{\kappa_s}\right) < \frac{\nu_\xi}{\kappa_s^2}(\kappa_s - \nu_\epsilon).$$

Turning to Part (ii) of the proposition, denote $a \equiv a_1 - a_2$. From Lemma 1, we have

$$\begin{aligned}
& \text{Cov}(P_1 - P_0, P_2 - P_1) \\
= \; & \text{Cov}\left(E_\ell(\theta|f), E_\ell(\theta|F) - E_\ell(\theta|f)\right) - (b_2\mu + a)a\nu_z,
\end{aligned}$$

and

$$\begin{aligned}
& \text{Cov}(P_2 - P_1, P_3 - P_2) \\
= \; & \text{Cov}\left(E_\ell(\theta|F) - E_\ell(\theta|f), E_\ell(\theta|s) - E_\ell(\theta|F)\right) - (c_3\mu + b_2)\, b_2\nu_z + (b_2\mu)a\nu_z,
\end{aligned}$$

and

$$\begin{aligned}
& \text{Cov}(P_3 - P_2, P_4 - P_3) \\
= \; & \text{Cov}\left(E_\ell(\theta|s) - E_\ell(\theta|F), \theta - E_\ell(\theta|s)\right) + (c_3\mu)\, b_2\nu_z.
\end{aligned}$$

It also follows that the short-run reversals parameter

$$\begin{aligned}
\mathcal{S} \; & \propto \; \text{Cov}(P_1 - P_0, P_2 - P_1) + \text{Cov}(P_2 - P_1, P_3 - P_2) + \text{Cov}(P_3 - P_2, P_4 - P_3) \\
& = \; \mathcal{S}^K - a^2\nu_z - b_2^2\nu_z, \qquad \text{(A.25)}
\end{aligned}$$

where $\mathcal{S}^K$ is defined in Equation (A.21), and $a \equiv a_1 - a_2$. It is straightforward to show after taking derivatives that $\mathcal{S}$ decreases in ν_z. It is obvious that $\mathcal{S} < 0$ under Condition (A.23). Further, note that $\mathcal{S}$ depends on μ only through a, and $d\mathcal{S}/da < 0$. Now,

$$\begin{aligned}
\frac{da}{d\mu} \; & \propto \; 2\left(b_2 + c_3\mu\right) c_3 \left(\frac{1}{\nu_z} + Ab_2 + Ac_3\mu^2\right) - 2Ac_3\left(b_2 + c_3\mu\right)^2 \mu \\
& \quad - \left[b_2\left(b_2 + 2c_3\mu\right)\left(\frac{1}{\nu_z} + Ab_2 + Ac_3\mu^2\right) - 2Ac_3\left(b_2 + c_3\mu\right) b_2\mu^2\right] \\
& = \; \left[2\left(b_2 + c_3\mu\right) c_3 - b_2\left(b_2 + 2c_3\mu\right)\right]\left(\frac{1}{\nu_z} + Ab_2 + Ac_3\mu^2\right) \\
& \quad -2Ac_3\left(b_2 + c_3\mu\right)\left(b_2 + c_3\mu - b_2\mu\right)\mu \\
& < \; \left[2\left(b_2 + c_3\mu\right) c_3 - b_2\left(b_2 + 2c_3\mu\right)\right]\left(\frac{1}{\nu_z} + Ab_2 + Ac_3\mu^2\right) \\
& \propto \; 2\left(b_2 + c_3\mu\right) c_3 - b_2\left(b_2 + 2c_3\mu\right) \\
& < \; 2\left(b_2 + c_3\mu\right) c_3 - b_2\left(b_2 + c_3\mu\right) \\
& \propto \; 2c_3 - b_2 \\
& = \; 2\kappa_{\theta|s} - \kappa_{E_\ell(\theta|s)|F},
\end{aligned}$$

where the first inequality obtains from $\mu \leq 1$, the second inequality obtains from $\mu \geq 0$, and the last equality obtains from the expressions of c_3 and b_2 in the proof of Lemma 1. Whenever $2\kappa_{\theta|s} < \kappa_{E_\ell(\theta|s)|F}$, $da/d\mu < 0$; so that $\mathcal{S}$ increases in μ.

For Part (iii) of the Proposition, set $\mu = 0$. Then, note from Equation (A.24) that $\mathcal{L}$ decreases in ν_z. Similarly, from Equation (A.25), $\mathcal{S} < 0$ decreases in ν_z (i.e., $\mathcal{S}$ becomes more negative as ν_z increases). Given that all endogenous parameters are continuous in μ, Part (iii) follows. □

Proof of Proposition 3: Note from the proof of Proposition 2 that

$$\mathcal{L}^* = \mathcal{L} - \text{Cov}(P_2 - P_0, P_3 - P_2),$$

where

$$\begin{aligned}
& \text{Cov}(P_2 - P_0, P_3 - P_2) \\
= \ & \text{Cov}\left(E_\ell(\theta|F), E_\ell(\theta|s) - E_\ell(\theta|F)\right) - (c_3\mu + b_2)b_2\nu_z - (b_2\mu)^2\nu_z \\
< \ & \text{Cov}\left(E_\ell(\theta|F), E_\ell(\theta|s) - E_\ell(\theta|F)\right) - b_2^2\nu_z \\
< \ & 0,
\end{aligned}$$

where the last inequality obtains under Condition (A.23).

From Lemma 1,

$$\mathcal{L}^* = \text{Cov}(P_2 - P_0, P_4 - P_3) = \text{Cov}\left(E_\ell(\theta|F), \theta - E_\ell(\theta|s)\right) - (c_3\mu + b_2)c_3\mu\nu_z$$

decreases in ν_z. □

Proof of Proposition 4: Note from Lemma 1 that

$$\begin{aligned}
\mathcal{S}_{(2)} &= \frac{\text{Cov}(P_1 - P_0, P_3 - P_2) + \text{Cov}(P_2 - P_1, P_4 - P_3)}{2} \\
&\propto \Big[\text{Cov}\left(E_\ell(\theta|f), E_\ell(\theta|s) - E_\ell(\theta|F)\right) + \text{Cov}\left(E_\ell(\theta|F) - E_\ell(\theta|f), \theta - E_\ell(\theta|s)\right)\Big] \\
&\quad -(b_2\mu + a)b_2\mu\nu_z - (c_3\mu + b_2)c_3\mu\nu_z.
\end{aligned}$$

It is straightforward to show that the value in the bracket is proportional to $\nu_\theta - \nu_{\theta_1} > 0$. It follows that if $\mu = 0$, then $\mathcal{S}_{(2)}$ does not depend on ν_z, and $\mathcal{S}_{(2)} > 0$. If $\mu > 0$, $\mathcal{S}_{(2)}$ decreases in ν_z (note that $a \equiv a_1 - a_2$ in Lemma 1 increases in ν_z), and b_2 and c_3 do not involve ν_z. Thus, as ν_z increases from zero, $\mathcal{S}_{(2)}$ starts positive, and eventually turns negative.

Also from Lemma 1,

$$\begin{aligned}
\mathcal{S}_{(3)} &= \text{Cov}(P_1 - P_0, P_4 - P_3) \\
&= \text{Cov}\left(E_\ell(\theta|f), \theta - E_\ell(\theta|s)\right) = \text{Cov}\left(\frac{\nu_{\theta_1}}{\kappa_f} f, \theta - \frac{\nu_{\theta_1}}{\kappa_s} s\right) \\
&= \frac{\nu_{\theta_1}}{\kappa_f}\left(\nu_\theta - \frac{\nu_{\theta_1}}{\kappa_s}\nu_s\right) \propto \nu_\theta - \nu_{\theta_1} > 0.
\end{aligned}$$

□

Proof of Proposition 5: Note from Lemma 1 that

$$\begin{aligned}
& \text{Cov}(P_2 - P_1, P_4 - P_3) \\
= \; & \text{Cov}\left(E_\ell(\theta|F) - E_\ell(\theta|f), \theta - E_\ell(\theta|s)\right) - (c_3\mu + b_2)c_3\mu\nu_z.
\end{aligned}$$

If $\mu = 0$, then

$$\text{Cov}(P_2 - P_1, P_4 - P_3) = \text{Cov}\left(E_\ell(\theta|F) - E_\ell(\theta|f), \theta - E_\ell(\theta|s)\right) > 0, \tag{A.26}$$

where the inequality obtains from noting that

$$\begin{aligned}
\text{Cov}\left(E_\ell(\theta|F) - E_\ell(\theta|f), \theta - E_\ell(\theta|s)\right) &= \text{Cov}\left(\frac{\nu_{\theta_1}}{\kappa_F} F - \frac{\nu_{\theta_1}}{\kappa_f} f, \theta - \frac{\nu_{\theta_1}}{\kappa_s} s\right) \\
&= \left(\frac{\nu_{\theta_1}}{\kappa_F} - \frac{\nu_{\theta_1}}{\kappa_f}\right)\left(\nu_\theta - \frac{\nu_{\theta_1}}{\kappa_s}\nu_s\right) \propto \nu_\theta - \nu_{\theta_1} > 0.
\end{aligned}$$

Also from Lemma 1,

$$\begin{aligned}
& \text{Cov}(P_2 - P_1, P_3 - P_2) \\
= \; & \text{Cov}\left(E_\ell(\theta|F) - E_\ell(\theta|f), E_\ell(\theta|s) - E_\ell(\theta|F)\right) - (c_3\mu + b_2)\, b_2\nu_z + (b_2\mu)a\nu_z.
\end{aligned}$$

If $\mu = 0$, then

$$\begin{aligned}
& \text{Cov}(P_2 - P_1, P_3 - P_2) \\
= \; & \text{Cov}\left(E_\ell(\theta|F) - E_\ell(\theta|f), E_\ell(\theta|s) - E_\ell(\theta|F)\right) - b_2^2\nu_z
\end{aligned} \tag{A.27}$$

and decreases in ν_z.

Also, from Lemma 1, since $0 \leq \mu \leq 1$,

$$a = a_1 - a_2 \geq A \frac{\nu_{\theta_1}^2}{\kappa_F} \left(1 - \frac{\kappa_F}{\kappa_f}\right).$$

For $a > \sqrt{3} b_2$, it suffices that

$$A \frac{\nu_{\theta_1}^2}{\kappa_F} \left(1 - \frac{\kappa_F}{\kappa_f}\right) > \sqrt{3} A (\nu_{\theta_1}^2 / \kappa_s)(1 - \kappa_s \kappa_F^{-1}),$$

which requires $\nu_\zeta / \kappa_f > \sqrt{3} \nu_\epsilon / \kappa_s$. In this case, it follows from the proof of Proposition 2 that

$$\mathcal{S} = \frac{\mathcal{S}^K - a^2 \nu_z - b_2^2 \nu_z}{3} < \frac{\mathcal{S}^K - 3 {b_2}^2 \nu_z - b_2^2 \nu_z}{3} < -b_2^2 \nu_z < \mathrm{Cov}(P_2 - P_1, P_3 - P_2),$$

where the second inequality obtains under Condition (A.23), and the last inequality obtains from Equation (A.27). Since all functions are continuous, the proposition follows. $\square$

Appendix B: An Infinite Horizon Setting

In this appendix, we extend our model in the main paper to an infinite horizon setting.[42] We further show that some assumptions in the main paper, specifically, normalizing the unconditional mean of the liquidation value V to be zero, and the return of the risk-free asset R_f to be one, are not required for our results on momentum and reversals.

The economy is divided into an infinite sequence of periods. There are five dates in a given period T: 0, 1, 2, 3, and 4. Date 4 of period T is also the beginning (Date 0) of period $T+1$. At Date 4 of every period T, the risky stock pays a dividend $d_T = \bar{d}_T + \theta_T$, where $\bar{d}_T$ is the unconditional mean of the risky asset's payoff and θ_T is a random normal variable (note that the main paper assumes a zero mean). The θ_T's at different periods are independent of each other. The signals and noise trades corresponding to a set of dates in a period T are subscripted by T and these are also i.i.d. across different periods.

In each period T, the i'th informed or uninformed investor trades at every date within that period, and consumes only at Date 4. The utility of consumption at Date 4 in period T is the standard exponential:

$$U(C_{i4T}) = -\exp(-A_c C_{i4T}),$$

where A_c is the risk aversion coefficient. We can use the standard "conjecture-verify" approach in dynamic programming (see, e.g., Wang (1994)) to show that the i'th informed or uninformed investor's value function at Date 4 takes the form:

$$J_\eta(W_{i4T}) \quad = \quad -\exp(-AW_{i4T} - Q_{\eta T}), \quad \text{and} J_\ell(W_{i4T}) = -\exp(-AW_{i4T} - Q_{\ell T}),$$

where $A = A_c(1 - R_f^{-4})$, and the Q's are constant parameters. We can use a similar derivation as in the proof of Proposition 1 in the main paper to show that the (ex-dividend)

[42] See also, for example, Holden and Subrahmanyam (2002) (Section II) for a similar exercise.

prices of the risky stock at Dates 0, 1, 2, and 3 can be expressed as:

$$\begin{aligned}
P_{3T} &= \frac{\bar{V}_T + \bar{d}_T + \gamma_1 \omega_T + \gamma_2 F_T + \gamma_3 \mu z_{2T}}{R_f}, \\
P_{2T} &= \frac{\bar{V}_T + \bar{d}_T + \beta_1 F_T + \gamma_3 \mu z_{2T} + \beta_2 (z_{2T} + \mu z_{1T}) - \beta_3 z_{2T}}{R_f^2}, \\
P_{1T} &= \frac{\bar{V}_T + \bar{d}_T + \alpha_1 f + \beta_2 \mu z_{1T} + \alpha_2 z_{2T}}{R_f^3}, \\
P_{0T} &= \frac{\bar{V}_T + \bar{d}_T}{R_f^4},
\end{aligned}$$

where $\omega_T \equiv s_T + \delta z_{3T}$, α's, β's, γ's, and δ are constants as given in the proof of Proposition 1 of the paper, and $\bar{V}_T$ represents the ex-dividend price at Date 4 of the Tth period, which is specified according to the recursive formula $\bar{V}_{T-1} = R^{-4}(\bar{V}_T + \bar{d}_T)$.

Consider the following contrarian investments:

- At Date 1, sell (buy) $R_f^3 |P_{1T} - R_f P_{0T}|$ shares of the stock if $P_{1T} - R_f P_{0T}$ is positive (negative), and liquidate this position at Date 2; invest the proceeds in the risk-free asset until Date 4. The expected profit (in excess of the risk-free interest rate) from this investment is

$$-R_f^5 \mathrm{Cov}(P_{1T} - R_f P_{0T}, P_{2T} - R_f P_{1T}).$$

- At Date 2, sell (buy) $R_f^2 |P_{2T} - R_f P_{1T}|$ shares of the stock if $P_{2T} - R_f P_{1T}$ is positive (negative), and liquidate this position at Date 3; invest the proceeds in the risk-free asset until Date 4. The expected profit from this investment is

$$-R_f^3 \mathrm{Cov}(P_{2T} - R_f P_{1T}, P_{3T} - R_f P_{2T}).$$

- At Date 3, sell (buy) $R_f |P_{3T} - R_f P_{2T}|$ shares of the stock if $P_{3T} - R_f P_{2T}$ is positive (negative), and liquidate this position at Date 4. The expected profit from this investment is

$$-R_f \mathrm{Cov}(P_{3T} - R_f P_{2T}, P_{4T} - R_f P_{3T}).$$

Note that here we scale up the earlier investments, and allow reinvestment in the risk-free asset; this ensures that the three investments produce payoffs of similar scales. The average profit from these contrarian investments can be expressed as:

$$-\frac{1}{3}\sum_{t=1}^{3}\left[R_f^{7-2t}\text{Cov}(P_{tT}-R_fP_{t-1,T},P_{t+1,T}-R_fP_{tT})\right].$$

It is straightforward to verify that this profit is opposite in sign to the short-term predictability measure in the main paper, $\mathcal{S}$ (see Equation (1)).

Also consider the following momentum investment: at Date 2, buy (sell) $R_f^2|P_{2T}-R_f^2P_{0T}|$ shares of the stock if $P_{2T}-R_f^2P_{0T}$ is positive (negative), and hold this position until Date 4. The expected profit (in excess of the risk-free interest rate) from this momentum investment can be expressed as:

$$R_f^2\text{Cov}(P_{2T}-R_f^2P_{0T},P_{4T}-R_f^2P_{2T}).$$

It is straightforward to verify that this profit is proportional to the long-term predictability measure in the main paper, $\mathcal{L}$ (see Equation (2)).

In sum, our analysis here shows that our model holds within an infinite horizon setting. In addition, normalizing the unconditional mean of the risky asset's payoff V to be zero and the gross risk-free interest rate R_f to be one (as in the main paper) is without loss of generality.